Defined contribution pensions, married couples, and the "annuity puzzle"*

Carlos VIDAL-MELIÁ**
Ana LEJÁRRAGA-GARCÍA
José E. DEVESA-CARPIO

Abstract: The aim of this paper is to look into the problem faced by married couples when they have to decide how to receive their pension. With this end in view, we set out an optimization model based on ideas first put forward by Brown & Poterba (2000). This model is then applied to determine the expected utility for the couple provided by programmed withdrawal, joint life annuity, and joint life annuity with a last survivor payout rule, according to the couple's possible preferences for consumption and risk perceptions. Finally, we carry out an analysis of the couple's welfare based on the concept of equivalent wealth with the aim of unravelling the so-called "annuity puzzle". Results are presented for a model specification calibrated to Spain.

Key words: programmed withdrawal, joint survivor annuity, lifetime annuity with contingent survivor benefit, utility.

JEL Classification: G23, H55, J26.

Resumen: El objetivo de este trabajo es abordar el problema al que debe enfrentarse, un jubilado casado o con pareja, respecto a la adopción de decisiones sobre la modalidad de cobro de su pensión. Con este objetivo, se desarrolla un modelo de optimización basado en el planteamiento inicialmente realizado por Brown & Poterba (2000), que se aplica para determinar la utilidad esperada que proporciona a la pareja el retiro programado, la renta vitalicia conjunta y la renta vitalicia reversible en función de sus distintas preferencias por el consumo y percepciones del riesgo. También se realiza un análisis del bienestar de la pareja a partir del concepto de riqueza equivalente. Finalmente, mediante la incorporación de diversas características al modelo básico que lo acercan a la realidad, se intenta arrojar luz sobre el "enigma de las rentas vitalicias". Los resultados se presentan para el caso español.

Palabras clave: Retiro programado, renta vitalicia conjunta, renta vitalicia reversible, utilidad.

Clasificación JEL: G23, H55, J26.

---

* The authors would like to thank Pierre Devolder, Inmaculada Domínguez-Fabián, Peter Hall, Julio Lucia-López, Salvador Valdés-Prieto and Manuel Ventura-Marco for their help. Preliminary versions of this paper have been presented at two international congresses in Portugal and Italy. Financial support from FUNCAS is gratefully acknowledged. Any errors the paper may contain are entirely the authors' responsibility.

** Facultad de Economía. Departamento de Economía Financiera. Universidad de Valencia. Avenida de los Naranjos, s.n. 46022 Valencia (Spain). Phone: + 34 963828369. Fax: + 34 963828370.
e-mail: carlos.vidal@uv.es (Corresponding author). e-mail: ana.lejarraga@gseguros.com
e-mail: enrique.devesa@uv.es

1.-INTRODUCTION

One of the main differences between pension plans, or defined benefit pension systems, and defined contribution pension schemes is the way the payout flow is distributed in retirement. Social Security and defined benefit schemes generally offer pensions in the form of lifetime annuities, providing pensioners with insurance against a lack of resources and the risk of longevity[1]. These annuities usually change in line with the retail price index (RPI), which means they retain their initial purchasing power over time. Defined contribution plans, on the other hand, provide an accumulated amount on retirement. This amount is the result of the capitalization of the separate contributions. In most cases the individual can receive this amount in a number of different ways according to personal choice, such as a lump sum, a fixed annuity or an indexed annuity, for a temporary period or for life, with or without any type of survivor benefit or guarantee of leaving money in the estate after death.

In this paper we look at the problems faced by married couples regarding the decisions they have to make as to how to receive their pension and determine optimal consumption in the retirement period, our aim being to throw some light on the so-called "annuity puzzle". As Brown & Poterba (2000) have pointed out, there is still a lot of research to be done in this area. In order to make a sound decision, couples have to take into account their life expectancy, their motivation with regard to leaving a bequest, and the possible existence of other annuities from Social Security amongst other things.

Unlike what happens in the context of individuals, one possible reason for the low demand for lifetime annuities is that couples can optimize their joint consumption path rather than access the life annuities market, and thereby gain substantially in terms of welfare

As is well known, lifetime annuity contracts transfer resources from those who die first to those who live longer. Something similar happens when the members of a family pool their income in such a way that, when one of them dies, the remaining members can make use of the wealth unconsumed up to that moment. As Brown (2000) has pointed out, it is clear that the more people there are in the family, the more this will resemble the transfer of resources in an insured group through annuity contracts. According to the World Bank (2001), the family can be described as an "incomplete" life annuities market since, in theory, even a small family unit can take unstructured measures that could generate up to three-quarters of the welfare gains of an actuarially fair life annuities market. More specifically, Kotlikoff & Spivak (1981) calculate that a two-person household with identical probabilities of survival could achieve almost 50% of the additional welfare reached in actuarially fair annuities markets.

Brown (2000) looks at the case of a family made up of several people. Given that the risk that all members of the family will die in one particular year is logically lower than the probability that one of them will die in that year, and that the amount of the annuity is inversely proportional to the probability of survival, then the return on a joint annuity is less than the return on individual annuities[2]. This makes lifetime annuities less attractive to families.

1 The risk of needing more resources through living longer than expected.

2 The higher the risk of death, the higher the amount of pension provided by the insurance company.

All the above points could lead one to think that what Friedman & Warshawsky (1990) and Brown (2001) term the "annuity puzzle"[3] is actually nothing of the kind. Indeed, a reasonable explanation as to why many individuals and couples do not buy annuities can be found when we take into account so-called "market imperfections", the existence of Social Security pensions in the form of indexed lifetime annuities, the inflexibility in the way benefits are paid, the different ways in which different types of pension are taxed in some countries (not always to the benefit of the lifetime annuity option), the capacity of families to pool risks and thereby substitute the private pensions market to some extent, and the inherent limitations of the models used, which leave some presumably important questions unanswered.

Another important reason why people often do not like annuities is the fact that the guaranteed interest rate involved in the price of the annuity seems very small in comparison to the short-term interest rates available on the market, for instance. People do not think about mortality rates or loading rates, but they can have an immediate idea about the interest rate. In Belgium some years ago[4], for example, annuities were computed at 4.75% while at the same time the short-term interest rate was 10%.

The most important new element this paper introduces is the extension of Brown & Poterba's model (2000) in at least four areas:

1.- It considers different values for the discount rate of the agents.

2.- It introduces joint programmed withdrawal, a type of benefit available in a number of Latin American countries.

3.- Apart from analyzing equivalent wealth, it also introduces another way of measuring the welfare provided by buying a lifetime annuity, calculating what percentage of the couple's initial wealth should be allocated to buying the pension so as to maximize the expected utility obtained from the optimal consumption path deriving from the income available at each moment.

4.- Valid recommendations are taken from political economy concerning the regulation of pension types in both compulsory and voluntary capitalization systems. The aspect regarding voluntary capitalization systems is very closely linked to the problems in Spain.

3 Following Vidal & Lejárraga (2004), the "annuity puzzle" is that empirical evidence shows the extreme rarity of voluntary private individual annuity contracts even though, according to Yaari (1965), individuals would be better off holding only annuitized assets in the absence of a bequest motive, or a portfolio of annuitized and traditional assets in the presence of a bequest motive. It must be emphasized that in Yaari's model the purchase of life annuities is optimal under the following assumptions:
1) Consumers maximize the expected (Neumann-Morgenstern) utility with separability and additivity. 2) The only risk faced by consumers is longevity risk. 3) There are no other assets, such as housing, that offer another source of annuity that is characteristically uncorrelated. 4) The individual is single and has no descendents. 5) The individual has no access to pensions in the form of life annuities from the first pillar of the social security system. 6) The annuity market is actuarially fair. 7) There is only one conventional asset which pays a given interest rate. 8) Consumers can borrow and lend at this same rate. Davidoff et al. (2003) show that the conditions under which the purchase of annuities is optimal are not as demanding as those set out by Yaari (1965). If financial markets are complete, one only requires that no bequest motive exists and that the expected rate of return on annuities is greater than that on a reference financial asset. Partial annuitization is optimal when the condition of complete insurance market is relaxed.

4 Pierre Devolder, personal communication, April, 6, 2004.

The structure of this paper is as follows. In the next section we set out an optimization model based on ideas first put forward by Brown & Poterba (2000). This is then used to determine the expected utility provided by each of the pension types analyzed according to a couple's various preferences and perceptions. The types of pension considered are joint lifetime annuity with a last survivor payout rule (or simply joint survivor annuity), lifetime annuity with contingent survivor benefit and joint programmed withdrawal. In the third section we carry out an analysis of the couple's welfare based on the concept of equivalent wealth. Finally, by incorporating a number of different features -such as "market imperfections" (Section 4) and the possibility that some of the wealth is already invested in preexisting lifetime annuities (Section 5)- to the basic model to make it a better reflection of reality, we extend our analysis of welfare with the aim of unravelling the "annuity puzzle" and seeing if the types of pension available in defined contribution systems should be limited exclusively to life annuities.

2.-THE BASIC MODEL

Joint utility functions need to be considered in order to measure the expected utility deriving from the optimization of consumption for a married couple. This paper concentrates on the case of a couple in which at least one of the members is of retirement age and in which there are no children to be included as dependents when buying a pension. This is a realistic assumption since there will not normally be any dependent children by this age.

When the time comes to retire, the couple have to decide how to distribute their accumulated wealth over time in order to ensure they will be able to cover their future consumption needs, and try to maximize the expected utility they will obtain with the consumption path they choose. It is assumed that the couple can allocate their entire wealth to one of the pension types described below, and that there are no bequest motives. Obtaining the optimal consumption path in this multi-period, stochastic lifecycle model (the uncertainty stems exclusively from not knowing the date on which the individual will die) calls for multi-period or dynamic optimization techniques to be applied. No other type of uncertainty with regard to the interest rate, the evolution of mortality or the rate of inflation is considered.

Let C_t^m and C_{t-n}^f denote the consumption of the husband and the wife respectively, at each age t, in which we consider the wife to be n years younger than the husband, although the notation will also be valid in the case where the wife is older than her husband simply by allocating a negative value to n. U_c represents the couple's utility function when both of them are alive, while U_m and U_f are the utility functions for the man after his wife has died, and for the woman after her husband is dead, respectively.

Following Brown & Poterba (2000), it is assumed that the household utility function is a weighted sum of the utility functions of both members of the couple, U_m and U_f. Specifically:

$$U_c\left(C_t^m, C_{t-n}^f\right) = U_m\left(C_t^m + \lambda C_{t-n}^f\right) + \varphi U_f\left(C_{t-n}^f + \lambda C_t^m\right) \quad [1.]$$

where parameter φ represents the relative weight of the wife's utility in the household utility aggregate.

The husband's utility function depends on ($C_t^m + \lambda C_{t-n}^f$) and the wife's on ($C_{t-n}^f + \lambda C_t^m$), where λ is the percentage of consumption that can be shared. This leads to the possibility that some goods (for example magazines, the family home, the car, etc…) can be "joint goods" in such a way that their purchase by one of the members of the couple means that the other member can consume or use them too. When λ=0 there is no joint consumption, and each individual's utility function shows only his or her own consumption. When λ=1, all consumption is joint and the consumption needs of both members of the family unit are equal.

2.1.- The couple does not have access to the lifetime annuities or programmed withdrawal market

In this case the problem of utility maximization would be the following:

$$\max_C \sum_{t=e_r}^{\infty} \frac{\overbrace{U_c(C_t^m, C_{t-n}^f)\,{}_{t+1-e_r}P_{e_r}^m \cdot {}_{t-n+1-y}P_y^f}^{1} + \overbrace{U_m(C_t^m, 0)\,{}_{t+1-e_r}P_{e_r}^m \cdot {}_{/t-n+1-y}q_y^f}^{2} + \overbrace{U_f(0, C_{t-n}^f)\,{}_{t-n+1-y}P_y^f \cdot {}_{/t+1-e_r}q_{e_r}^m}^{3}}{(1+\delta)^{t+1-e_r}} \quad [2.]$$

$$\text{s.t.} \quad C_t^m + C_{t-n}^f = W_t(1+r)(1+\pi) - W_{t+1} \quad [3.]$$

$$W_t \geq 0, \quad \forall\, t > e_r \quad [4.]$$

where:

δ: The pure time preference rate, i.e. the classic exponential discount factor for future utility.

e_r: Retirement age.

W_t: Wealth corresponding to age *t* for the man and age *t-n* for the woman.

W_{er}: Wealth corresponding to age e_r for a man and age e_r*-n* for a woman, equal to the initial wealth at retirement, W_0.

r: Real expected risk-free rate of return (assumed to be constant throughout the couple's lifetime).

y: The woman's age at the initial moment, which will be equal to $e_r - n$.

${}_{t+1-e_r}P_{e_r}^m$: Probability that a man aged e_r will live for another $t+1-e_r$ years.

${}_{t-n+1-y}P_y^f$: Probability that a woman aged *y* will live for another *t-n+1-y* years.

${}_{/t+1-e_r}q_{e_r}^m$: Probability that a man aged er will die before living another $t+1-e_r$ years.

${}_{/t-n+1-y}q_y^f$: Probability that a woman aged y will die before living another t-n+1-y years.

π : Expected rate of inflation.

It is assumed that the husband's and the wife's probabilities of dying are independent. The importance of the effect of dependent mortality on annuity valuation is not very clear in the literature. Frees *et al.* (1996) find that annuity values are reduced by approximately 5% when dependent mortality models are used compared to the standard models that assume independence, whereas Brown & Poterba (2000) report only modest "broken heart"[4] effects on the annuity equivalent wealth measure. In Spain, as in most countries, standard insurance industry practice assumes independence of lives when valuing annuities where the promise to pay is based on more than one life.

The three possible valid states for consumption are shown along with their associated probability in brackets 1, 2 and 3 in the numerator of Equation 2. The first state is the assumption that both are still alive; in the second only the man is alive; and finally in the third state only the woman is alive. There is a possible fourth state - in which both have died - but logically there would be neither consumption nor utility in this case.

2.2.- The couple have access to the joint annuity with contingent survivor benefit, joint survivor annuity and programmed withdrawal market

Here we analyze two alternative lifetime annuities which enable the mortality risk of each member of the couple to be transferred, and a type of pension called programmed withdrawal.

1) ***Lifetime annuity with a contingent survivor benefit***, a percentage of which becomes payable to a designated beneficiary, normally the surviving spouse (in general terms this is assumed to be the woman). With this type of lifetime annuity a periodic payment is made to the primary annuitant, which he receives until his death. From this moment his spouse, assuming she has survived until this date, will start to receive an amount calculated as a percentage of the amount the deceased annuitant was receiving. This percentage is set by the purchaser when the relevant insurance policy is signed. The initial amount of pension - in the case where the primary annuitant is the man - will be determined by:

$$A_{e_r} = \frac{W_{ANNUITY}}{\sum_{t=e_r}^{\infty} \frac{(1+\alpha)^{t-e_r} \cdot \theta \cdot \left[{}_{t+1-e_r}P_{e_r}^{m*} + \gamma \; {}_{t-n+1-y}P_y^{f*} (1 - {}_{t+1-e_r}P_{e_r}^{m*}) \right]}{\left[(1+r)(1+\pi)\right]^{t-(e_r-1)}}} \qquad [5.]$$

$$A_t = A_{e_r} (1+\alpha)^{t-e_r}, \qquad \forall t > e_r \qquad [6.]$$

This is a reversionary life annuity with the woman as beneficiary. It increases by an annual accumulative increment equal to α. If the pension is indexed to inflation, α will coincide with π. When $\alpha=0$, the pension is constant, i.e. decreasing in real terms. The other variables involved are:

5 The tendency is for the mortality rates of surviving spouses to be somewhat higher for several years after their spouse's death than the mortality rates for similar individuals who have not lost a spouse.

γ: Percentage payable to the designated beneficiary.

θ: Management costs charged by the insurance company, normally $\theta = \left[\frac{1+ggi}{1-gge} \right]$, where ggi is the charge set by the company to cover the administration costs of the insurance policy, and gge is the external management charge applied to cover costs arising from the marketing of the policy.

${}_{t+1-e_r}P_{e_r}^{m*}$: Probability that an individual aged e_r will reach age t+1, according to the mortality tables used by the insurance company.

${}_{t-n+1-y}P_{y}^{f*}$: Probability that a woman aged y will live for another t-n+1-y years, according to the mortality tables used by the insurance company.

$W_{ANNUITY}$: The portion of initial wealth that the individual allocates to the purchase of the lifetime annuity.

It has also been assumed that the level of pension to which the couple have access when they buy the reversionary annuity is greater than the guaranteed minimum pension set by some pension systems.

It has also been considered in the basic model that the real technical interest rate that the insurance company guarantees on the annuity coincides with real market interest rate r.

2) ***Joint survivor life annuity***, which is a contract whereby the insurance company undertakes to pay a periodic amount while both members of the couple are alive, and a fraction of this amount, r or h, when one of them has died, for as long as the other lives. This fraction is normally 1, 2/3 or 1/2.

In this case the initial term of the annuity would be:

$$A_{e_r} = \frac{W_{ANNUITY}}{\sum_{t=e_r}^{\infty} \frac{(1+\alpha)^{t-e_r} \cdot \theta \cdot \left[{}_{t+1-e_r}P_{e_r}^{m*} \cdot {}_{t-n+1-y}P_{y}^{f*} + \rho\, {}_{t+1-e_r}P_{e_r}^{m*} (1 - {}_{t-n+1-y}P_{y}^{f*}) + \eta\, {}_{t-n+1-y}P_{y}^{f*} (1 - {}_{t+1-e_r}P_{e_r}^{m*}) \right]}{\left[(1+r)(1+\pi) \right]^{t-(e_r-1)}}} \quad [7.]$$

$$A_t = A_{e_r} (1+\alpha)^{t-e_r}, \qquad \forall t > e_r \quad [8.]$$

In the case where $\eta=\rho=100\%$, the amount of this pension would coincide with the amount of the annuity with contingent survivor benefit determined following Equation (5), with a reversionary percentage of $\gamma=100\%$.

When $\eta=\rho=50\%$, the joint survivor life annuity would be equivalent to the case where both members of the couple are entitled to a retirement pension of the same amount until they die.

As in the previous case, it has been assumed that the level of pension to which the couple have access when they buy the joint survivor annuity is greater than the guaranteed minimum set by some pension systems.

The annuities most usually offered on the market are those with a contingent survivor benefit rule, although in some countries it is not unusual for joint survivor annuities to be bought.

3) ***Programmed withdrawal*** is a type of benefit available in a number of Latin American countries. As explained by Devesa & Vidal (2001), with programmed withdrawal the pensioner receives his pension charged on the balance in his individual capitalization account (ICA). This account remains under the administrator's responsibility and management, and so enables the retiree to benefit from the return on the fund. In Chile, for example, the pension is set for annual periods and expressed in "unidades de fomento", a reference unit adjusted daily in line with the RPI. The amount is calculated annually by taking into account the balance in the individual account, the technical interest rate defined by law, and the life expectancy of the pensioner and his family unit according to the relevant mortality tables. In other words, the value of the periodic pension the retiree will receive depends on the wealth he has accumulated in his individual capitalization account at each moment. Basing its calculations on this accumulated wealth, the insurance company or pension fund administrator will determine the amount of the pension to be withdrawn, setting the equivalence with the actual actuarial value of a lifetime annuity with a reversionary percentage in favour of the surviving spouse.

Thus the maximum amount of pension that can be withdrawn at each age t (in the case where the primary annuitant is the husband) is given by:

$$A_t = \frac{W_t^{PW}(1+i)(1+\pi)}{\sum_{s=t}^{\infty} \frac{(1+\alpha)^{s-t}\cdot\theta\cdot\left[{}_{s+1-t}P_t^{m*} + \gamma\cdot {}_{s-n+1-(y+(t-e_r))}P_{y+(t-e_r)}^{f*}\cdot(1-{}_{s+1-t}P_t^{m*})\right]}{\left[(1+i)(1+\pi)\right]^{s+1-t}}}, \forall t \qquad [9.]$$

where W_t^{PW} is the fund accumulated in the capitalization account at the start of the annual period corresponding to age t:

$$W_t^{PW} = W_{t-1}^{PW}(1+i)(1+\pi) - A_{t-1}, \qquad \forall t \qquad [10.]$$

i is the return expected from the fund, which in principle is assumed to be equal to the return actually achieved, and γ is the percentage payable to the surviving spouse. In this case the amount of pension is decreasing over time and, unlike lifetime annuities, programmed withdrawal does not insure the couple against the risk of longevity. It could be said that programmed withdrawal is an instrument calling for financial discipline and the avoidance of an excessively rapid consumption of the accumulated resources.

When the couple decide to allocate part of their initial accumulated wealth to buying a joint survivor annuity, an annuity with a contingent survivor benefit or programmed withdrawal, the model for utility optimization can be expressed as:

$$\text{s.t.} \quad C_t^m + C_{t-n}^f = W_t(1+r)(1+\pi) - W_{t+1} + A_t \qquad [11.]$$

$$W_t \geq 0, \quad \forall\, t > e_r \qquad [12.]$$

where the term A_t, included in the budget constraint, represents the amount the couple will receive at moment t, deriving from the lifetime annuity contract. The value of this will be given by Equations (5) and (6) in the case of an annuity with a contingent survivor benefit, by (7) and (8) for a joint and survivor life annuity, and by (9) and (10) for programmed withdrawal with a last survivor payout.

2.3.-The couple have pre-existing lifetime annuities with a contingent survivor benefit

In the case of a couple who already have part of their wealth in a pre-existing lifetime annuity and decide to buy another one, the optimization model they would be faced with -assuming they do not have access to actuarially fair annuity markets- has the following constraints:

$$\text{s.t.} \quad C_t^m + C_{t-n}^f = W_t(1+r)(1+\pi) - W_{t+1} + R_t \qquad [13.]$$

$$W_t \geq 0, \quad \forall\, t > e_r \qquad [14.]$$

where:

$$W_{er} = W_{NP}, \qquad [15.]$$

and

$$W_0 = W_{NP} + W_{PA}, \qquad [16.]$$

and:

W_{NP}: Level of initial wealth not allocated to pensions.

W_{PA} :Level of initial wealth in pre-existing lifetime annuities.

R_t is a lifetime annuity with a 50% survivor payout, payable in arrears, index-linked to the RPI, assumed to derive from a pre-existing public or private pension system, obtained as:

$$R_{e_r} = \frac{W_{PA}}{\sum_{t=e_r}^{\infty} \frac{(1+\alpha)^{t-e_r} \cdot \left[{}_{t+1-e_r}P_{e_r}^m + \gamma \cdot {}_{t-n+1-y}P_y^f \cdot (1 - {}_{t+1-e_r}P_{e_r}^m)\right]}{\left[(1+r)(1+\pi)\right]^{t-(e_r-1)}}} \qquad [17.]$$

$$R_t = R_{er}(1+\pi)^{t-er}, \qquad \forall t \quad > e_r \qquad [18.]$$

Assuming there exists the possibility of buying actuarially fair annuities with a contingent survivor payout, in this context of there being a pre-existing annuity the optimization model represented in Equation 2 would have the following constraints:

$$\text{s.t.} \quad C_t^m + C_{t-n}^f = W_t(1+r)(1+\pi) - W_{t+1} + R_t + A_t \qquad [19.]$$

$$W_t \geq 0, \quad \forall\, t \ > e_r \qquad [20.]$$

where W_{er} is equal to the initial wealth not invested in annuities, i.e.:

$$W_{er} = W_{NR} \quad [21.]$$

and, $$W_0 = W_{NP} + W_{PA} + W_{ANNUITY} \quad [22.]$$

A_t is determined from Equation (5).

2.4.-The utility function and the optimization method

In order to obtain results, the analytical expressions of the utility functions used are the following:

$$U_m\left(C_t^m + \lambda C_{t-n}^f\right) = \begin{cases} \dfrac{\left(\dfrac{C_t^m + \lambda C_{t-n}^f}{(1+\pi)^{t+1-er}}\right)^{1-\beta} - 1}{1-\beta}, \beta \neq 1 \\ \log\left(\dfrac{C_t^m + \lambda C_{t-n}^f}{(1+\pi)^{t+1-er}}\right), \quad \beta = 1 \end{cases} \quad [23.]$$

$$U_f\left(C_{t-n}^f + \lambda C_t^m\right) = \begin{cases} \dfrac{\left(\dfrac{C_{t-n}^f + \lambda C_t^m}{(1+\pi)^{t+1-er}}\right)^{1-\beta} - 1}{1-\beta}, \beta \neq 1 \\ \log\left(\dfrac{C_{t-n}^f + \lambda C_t^m}{(1+\pi)^{t+1-er}}\right), \quad \beta = 1 \end{cases} \quad [24.]$$

In other words, just as is usually done in the case of an individual, we assume that the husband and wife have constant relative risk aversion (CRRA) utility functions, where $\beta > 0$ represents the risk aversion coefficient (Pratt-Arrow). A high β value means that the couple are less willing to substitute consumption over time in exchange for incentives supplied through the interest rate.

It is assumed that the degree of risk aversion is the same for both members of the couple. This is the assumption normally used in the literature and is also the most coherent, at least for the case we are dealing with, in which there is only a three-year age difference between the two. It is also fair to suppose that, over the years spent living together; the differences between their attitudes to risk will narrow[6].

The expected rate of inflation is p. As inflation is included, the consumption path is measured in real terms. This is one of the specification differences with regard to the paper by Brown

6 Halek & Eisenhauer (2001), Powell & Ansic (1997) and Jianakoplos & Bernasek (1998) among others have pointed out that women are significantly more risk averse than men, but we take the widespread assumption that both men and women have the same risk aversion coefficient.

& Poterba (2000). Obviously the risk of inflation is not taken into account since it is not accepted that the real rate of inflation could be different from the expected rate. Brown et al. (2001) considered the impact of inflation on the value of nominal annuities. They found that the inflation protection offered by a real annuity had only modest value. The wealth equivalent of nominal annuities decreased only slightly when they assumed i.i.d. inflation calibrated to 1926-97 data from the USA. When they assumed that inflation followed an AR(1) process the wealth equivalent further decreased, but the difference was only substantial at high coefficients of risk aversion or when the individual had no pre-annuitized wealth.

It is also assumed that there are no bequest motives, which means that mathematically no minimum level of wealth is required at any particular moment.

With this model, one possible solution for finding the optimal consumption path is to use multi-period optimization techniques. The mathematical models in this paper have been translated into LINGO® software programming language, and this program was used to obtain the numerical results shown in the various tables. For non-linear programming problems, the LINGO® package uses an algorithm based on the Generalized Reduced Gradient (GRG2) method. In addition to this, to help obtain a first feasible solution quickly it includes a recursive linear programming algorithm. GRG2 is based on Wolfe's reduced gradient method, later taken up by Abadie and Carpentier, in which the feasible improvement direction is not the generalized reduced gradient (GRG), but a second order approximation[7].

The objective function is included in Equation (2); the dynamic system is that formed by the difference equations that appear in the so-called budget constraint, i.e. Equations (3), (11), (13) and (19) depending on the case; the boundary conditions are: W_{er}=1 in the case of no annuities when no previous lifetime annuities exist, W_{er}=0.5 in the same case but when the couple have half their wealth in a pre-existing annuity, and W_{er}=0 when the individual has access to an annuity or programmed withdrawal; W_t are the state variables and C_t are the control variables; the state constraints are those expressed in Equations (4) or (12).

Brown & Poterba (2000) use dynamic programming to solve the problem. They say that analytic solutions can be found in some very simple cases, but that introducing constraint (4) means that "closed solutions" cannot be found.

2.5.-The couple's optimal consumption path for each type of pension

To determine the optimal consumption path we have considered that the type of pension bought is a lifetime annuity with a contingent survivor benefit payable at 50%, a programmed withdrawal with a 50% survivor payout (i.e. the value of γ=0.5), or a joint survivor annuity where on the death of either member of the couple, the surviving spouse would receive 50% of the pension payable when they were both alive (i.e. the value of ρ=η=0.5). Initially we have also considered the level of joint consumption (l) to be equal to 0, and the weighting factor of the woman's utility function to be φ=1.

To begin with we have considered that the insurance company applies no charges whatsoever on the purchase of the single premium life annuity (whether indexed or non-indexed), and also

7 This is a well-known algorithm that can be seen in detail in the papers by Ventura et al. (2000) and Bazaraa et al. (1993).

that the probabilities it uses to determine the premium (GRMF-95 mortality and survival tables[8]) coincide with the subjective probabilities of the consumer (those he considers when he evaluates expected utility according to the additional information he has). It has also been assumed that the real market interest rate, *r*, is 3%; that the degree of risk aversion can take one of the three values that adequately represent individuals; that the retirement age is 65 for men; that the wife is three years younger; and that inflation is equal to "*r*/2=1.5%"[9]. In addition, the following preference rate values have been set according to the individual's degree of impatience in accordance with the formula $\delta = \phi[(1+r)(1+\pi)-1]$ in which the value of ϕ (2, 1.5, 1, 0.5 and 0.25) qualifies individuals as (A): very impatient, (B): impatient, (C): indifferent, (D): slightly impatient, and (E): very slightly impatient. According to Yagi & Nishigaki (1993), the degree of the time discount rate is correlated to the degree of myopia, and this has an important effect on the demand for life annuities.

According to Valdés-Prieto (2002), in economic literature the very impatient individual, i.e. one with a relatively high preference for present over future consumption, is usually referred to as myopic. However, a distinction should be made between myopia and lack of foresight. Someone who suffers from myopia can predict the future just as well as someone who does not; he is fully aware that if he follows his preference for present consumption he will have to be content with a low level of consumption in the future.

Most of the papers quoted do not usually stress the degree of impatience. Seldom are impatient or very impatient people considered, possibly because they are more difficult to calculate. This is another specification difference with regard to the paper by Brown & Poterba (2000). These authors only appear to be interested in case C and do not calculate results for the rest. Nevertheless, as will be appreciated in the course of this paper, it is important to take into account a wide spectrum of attitude to consumption because of the variation in results.

Figures 1 and 2 show the optimal consumption path for different cases calculated at the time of retirement, which is when the decision has to be taken as to which type of pension is the most suitable. They show the optimal path the couple's consumption should follow from age 65 to 110[10] so as to maximize utility. Although the graphs reflect consumption at each age (nominal or real), it should be remembered that the weight consumption in advanced age represents for the

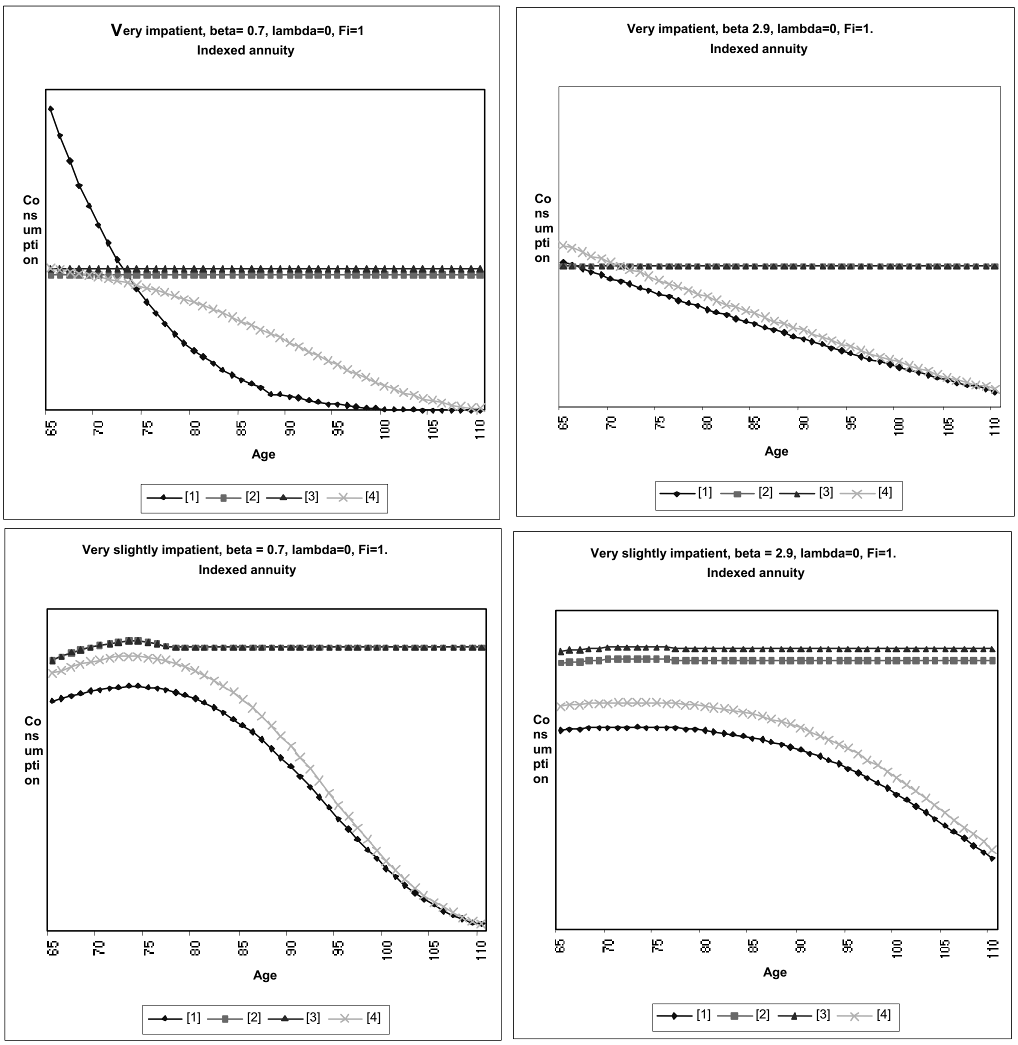
8 These tables are the ones normally used by insurance companies operating in Spain and, in general terms, they show a life expectancy for any particular age that is greater than that given in the latest tables available for the general population of Spain (mortality tables for the population of Spain 1998-1999 published by the National Institute of Statistics). One of the difficulties the company faces when determining the price it should charge for the annuities it sells is to predict accurately the mortality rates of the group to be insured. Obviously, if the company underestimates the evolution of mortality in the insured group, there will not be enough in the accumulated funds to cover payment of the annuities taken on, and the insurance company itself may become insolvent. To offset this - and taking into account the fact that the people who think they will live longer are those who demand more annuities - the insurance companies use specific mortality and survival tables for the groups of people who purchase annuities which are not the same as the mortality tables used for the general population.

9 The inflation rate used is a general estimate assigned by various macroeconomic forecasts to long-term inflation in Spain, although it is true that over the last few years real inflation has been higher. In addition to this, the interest rate used refers to the nominal technical interest rate which, for the data handled in the model (1.03*1.015)-1=0.04545, is approximately the long-term technical interest rate the insurance companies have used when selling annuities in Spain over the last two years.

10 Although calculations have been made up to the upper age limit on the mortality tables, 125 years, we have only shown up to 110 years because the last 15 years contribute nothing relevant and obscure a clear view of the most significant elements in the graphs.

Figure 1

Optimal consumption path, measured in real terms, for couples. Indexed annuities. [1]- Without annuity. [2]- Lifetime annuity with a contingent survivor benefit.[3]- Joint survivor annuity. [4]- Programmed withdrawal.



utility function is naturally very small because the probabilities of survival become lower and the discount factor reduces the present value more and more. It should also be remembered that these are not the only variables that interact in the model.

Clearly the optimal path will not necessarily be the same if it is recalculated years after first receiving the pension, since the probabilities of survival or the values of other variables may have changed.

Figure 2

Optimal consumption path, measured in nominal terms, for couples. Indexed annuities. [1]- Without annuity. [2]- Lifetime annuity with a contingent survivor benefit.[3]- Joint survivor annuity. [4]- Programmed withdrawal.

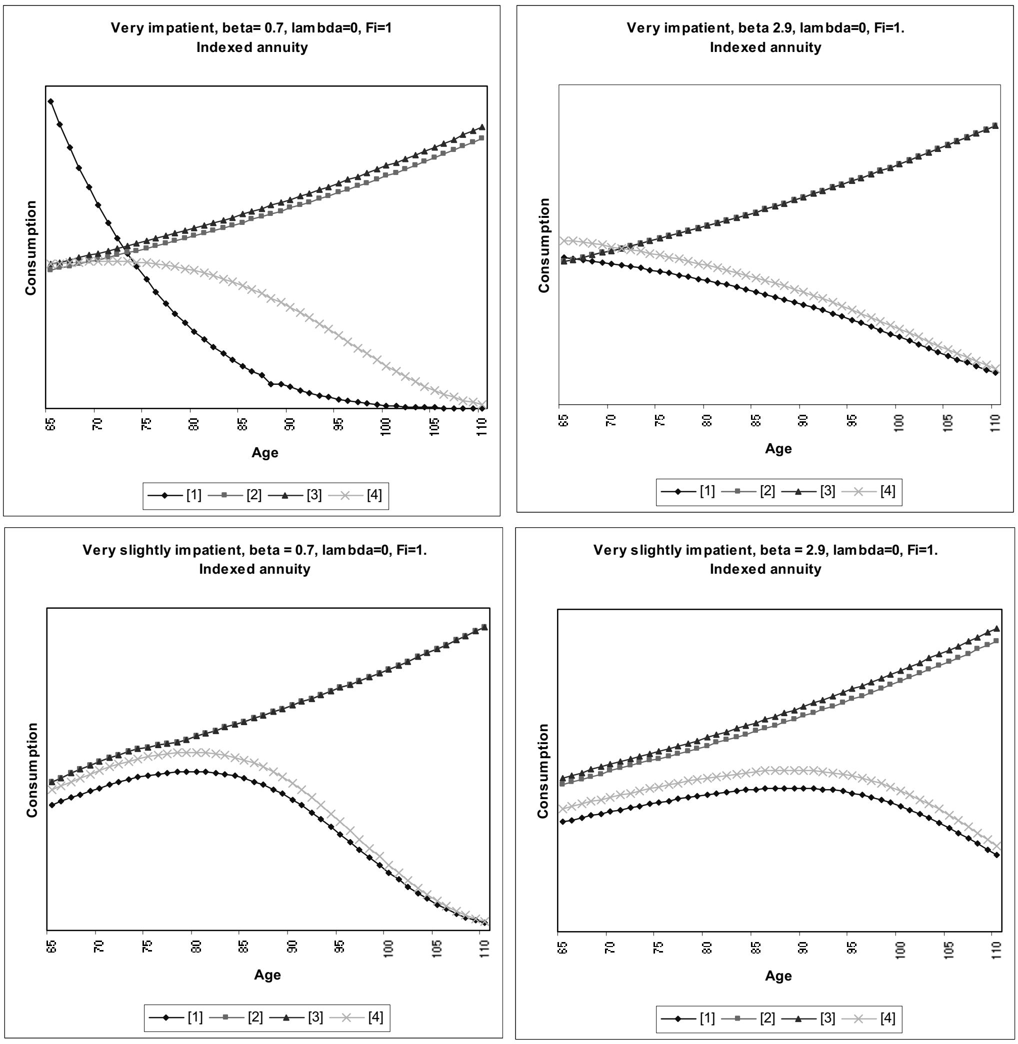


Figure 1 shows the optimal consumption path in real terms for the case where b takes on values 0.7 and 2.9, for a couple who are very impatient to consume and for another who are very slightly impatient. It can be seen that the behaviour of a couple that share consumption when buying a lifetime annuity is practically the same whether the pension is with a contingent survivor benefit or a joint survivor annuity. Consumption is always slightly greater in the case of the joint survivor life annuity. This is because the amount of the joint annuity is always slightly higher than the amount for an annuity with a contingent survivor payout since, in the former, when either member of the couple dies, in the case of a 50% survivor payout, the insurance company will only pay half of the initial pension, while in the latter the pension will only be reduced by 50% if the person who dies is the primary annuitant, the amount remaining unaltered on the death of the beneficiary.

However, a sizeable variation can be seen in the case of "no annuities". Consumption decreases noticeably during the first years when considering a couple who are very slightly impatient and for a value of β=0.7 in comparison to a very impatient couple, even when modifying the convexity of the function.

As is to be expected given the legal limitations on disposable income for consumption, the variation is less obvious in the case of programmed withdrawal (the return on the individual account is assumed to be 4% for the purpose of making up the graphs). For β=0.7, the range varies from a decreasing function for very impatient couples to one that grows at the beginning to reach its maximum at 73 years and then begins to drop gradually in the case of those who are slightly impatient (Gauss bell-shaped). Where β=2.9, the consumption path for programmed withdrawal is similar, and for very impatient couples consumption under programmed withdrawal is higher until age 70 than it would be in the case of joint annuities or annuities with contingent survivor benefits.

Figure 2 shows the optimal consumption path in nominal terms (as is normal in the literature) for the same cases as in Figure 1. The usual effect comes about when passing from a function in real terms to another in nominal terms with an expected rate of inflation constant for all periods; hence the joint annuities and annuities with survivor benefits which in Figure 1 were constant become increasing in nominal terms. In all the other cases there is a slight upward movement of the function.

According to Vidal et al. (2002), the consumption path behaves in a similar way when there is no access to the annuities market as it does in the case of an individual. For an individual who is very impatient and only slightly averse to risk, consumption is high at the start but rapidly decreases as the available resources diminish. As risk aversion increases and impatience for consumption decreases, the optimal consumption path - in the case where no wealth is allocated to buying the pension - becomes more stable, with less consumption at the start to enable more consumption later on. The optimal consumption path is always lower than those obtained with the purchase of lifetime annuities, whether joint survivor or with a contingent survivor payout.

3.-THE COUPLE'S EQUIVALENT WEALTH

One of the most usual ways of evaluating gains in welfare is to measure what level of wealth would be necessary for them to take on the same expected utility curve in any of the cases analyzed. This measure is *equivalent wealth*, given by:

$$\mu = \frac{W_0 + \Delta W}{W_0} = 1 + \frac{\Delta W}{W_0} \qquad [25.]$$

ΔW being the amount of additional wealth the couple would have to be given - following their optimal consumption path in any of the cases put forward - to enable them to reach the same level of utility they obtain when the consumption path is maximized in any of the others. The quotient $\Delta W/W_0$ determines the welfare gain for the couple, as a percentage of the level of wealth accumulated at the start. More specifically, this measure is aimed at determining by how much the couple, averse to risk, would value the possibility of being able to buy a lifetime annuity or programmed withdrawal as appropriate, and be able to protect themselves against the risk of excessive longevity in terms of the metric which the theory of utility supplies, in which both financial and psychological parameters such as their attitude to risk and consumption are taken into account.

As Valdés & Edwards (1998) point out, the evaluation will be an excess approach since it is considered that there is no other family insurance implicit, and neither are there other investments such as a house to offer another source of income not usually correlated.

3.1.-Equivalent wealth and lifetime annuity contracts

Using the same data and assumptions applied in the subsection on optimal consumption, in this section we calculate the value of equivalent wealth for couples and compare it with that obtained for individuals in the paper by Lejárraga et al. (2002). The meaning of equivalent wealth in this case is the amount of current wealth the couple would need in the absence of actuarially fair annuity markets in order to reach the same level of utility that they receive when such markets are available.

The results shown in Table 1 are those obtained in the hypothetical case of a couple - in which the male, the primary annuitant, is 65 years old on retirement with a spouse three years younger - who buy a lifetime annuity with a 50% contingent survivor benefit. A comparison with the case of an individual is also shown for the various parameters studied. In both parts of the table (A and B) the values for equivalent wealth of non-indexed (indexed) pensions that are gre-

Tabla 1

Equivalent wealth. Lifetime annuity with a 50% contingent survivor benefit. Comparison with a non-indexed lifetime annuity (man)

δ↓	β→	$\beta = 0.7$		$\beta = 2.9$		$\beta = 4.4$	
		LA50%CSP	**Individual**	**LA50%CSP**	**Individual**	**LA50%CSP**	**Individual**
A) Non-indexed annuity.							
A		**0.997**	1.092	1.281	1.448	1.354	1.546
B		1.094	1.190	1.325	1.500	1.380	1.581
C		*1.193*	1.292	1.360	1.545	1.400	1.609
D		1.279	1.390	1.386	1.581	1.416	1.631
E		1.312	1.433	1.397	1.596	1.423	1.641
B) Indexed annuity.							
A		**0.943**	1.049	1.225	1.406	1.308	1.515
B		1.054	1.159	1.293	1.480	1.363	1.577
C		1.176	1.281	1.359	1.555	1.416	1.638
D		1.297	1.407	1.422	1.628	1.465	1.697
E		1.350	1.467	1.449	1.662	1.487	1.724

ater than those of the indexed (non-indexed) pensions are underlined. Values lower than the unit, which mean that it is better not to allocate wealth to the purchase of lifetime annuities, appear in bold.

The result shown in italics in Table 1 means that for the couple there would be no difference between one monetary unit allocated to annuities and 1.193 units of wealth with no allocation to pensions. In other words, the couple would be willing to give up 1.193 units of ordinary wealth in order to have 1 monetary unit invested in a lifetime annuity with a 50% contingent survivor payout (LA50%CSP).

In all cases it can be seen that equivalent wealth grows when the degree of risk aversion increases and impatience decreases, just as happened in the case of a man who allocates all his wealth to the purchase of an individual lifetime annuity. Also, when the couple's joint consumption in the utility function is considered, the equivalent wealth is always lower than their consumption taken individually. This result was to be expected since the life expectancy of the couple is always greater than that of either of the two individuals taken separately.

For a fixed attitude towards consumption, however, the differences between equivalent wealth figures diminish as the degree of risk aversion grows. Thus for example in Table 1, part B, a consumer who is indifferent to impatience (C) and has a risk aversion degree of 0.7 obtains an almost 60% greater increase in welfare than a couple with the same characteristics[11]. The results for risk aversion degrees of 2.9 and 4.4 are 55% and 53% respectively. In the case of consumers who are very slightly impatient, the increases in welfare would vary noticeably, being 33%, 47% and 49% for the risk aversion values considered here.

Another similarity to the case of an individual is that when the couple are impatient for consumption they prefer to buy a non-indexed annuity, but choose an indexed pension when impatience for consumption decreases. If they are indifferent to consumption, the choice of pension type will depend on the degree of risk aversion they show: the greater their aversion to risk, the more useful it will be for them to buy an indexed lifetime annuity.

The rate of inflation in all the calculations has been taken as constant and linked to the real market interest rate (the former is assumed to be equal to half the latter). The results obtained are different if this parameter is modified. For example, if inflation were considered to be two times greater than the real market interest rate, and supposing all the other variables remained unchanged (specifically, the individual's degree of impatience is the same as that in the formula with 1.5% inflation), the preference for non-indexed pensions over indexed pensions decreases for almost all profiles. Only in the case of couples who have very little aversion to risk and a great deal of impatience is there greater welfare with level pensions.

It is worth pointing out that the expected utility and equivalent wealth obtained in the case of indexed pensions is independent of the inflation level considered. With non-indexed pensions, when there is a sharp increase in the rate of inflation, the level of equivalent wealth is very similar when risk aversion increases. The way the values for equivalent wealth draw closer to each other can be appreciated more clearly by considering an extremely high rate of inflation. For a

11 Table 1, part B): Increase in welfare with Beta=0.7 and indifferent to impatience in couples 17.6%; increase in welfare with Beta=0.7 and indifferent to impatience in individuals 28.1% therefore: ((28.1/17.6)-1)*100=59.66%.

Table 2

Percentage of initial wealth allocated to a lifetime annuity with 50% contingent survivor benefit which maximizes expected utility

δ↓	β→	Non-indexed annuity		Indexed annuity	
		$\beta = 0.7$	$\beta = 2.9$	$\beta = 0.7$	$\beta = 2.9$
A		52.76%	96.57%	41.81%	82.40%
C		97.21%	100.00%	87.67%	96.24%
E		100.00%	100.00%	100.00%	100.00%

rate of inflation equal to 8 times the interest rate, for example, the value of equivalent wealth is equal to 1.045 for whichever degree of risk aversion.

Another way of measuring the welfare provided by buying a lifetime annuity with a contingent survivor payout is to determine what percentage of the couple's initial wealth should be allocated to buying the pension so as to maximize[12] the expected utility obtained from the optimal consumption path deriving from the income available at each moment. This method is based on that applied by Friedman & Warshawsky (1990) in the case of the individual.

If we consider the possibility of buying a lifetime annuity with 50% payable to the surviving spouse, with the same assumptions considered in the calculations made for Table 1, the percentages showing what portion of the initial wealth (assuming there is no wealth invested in a preexisting pension) allocated to buying pensions will bring about maximum utility for the couple are shown in Table 2.

The results are most revealing: some couples would prefer not to allocate all their wealth to buying the lifetime annuity, i.e. they would achieve greater welfare gains by being able to chose what percentage they allocate to buying the lifetime annuity.

This method of measuring the additional welfare achieved through access to the actuarial pension market has a disadvantage with respect to measuring equivalent wealth in that it does not enable us to determine in each case whether it is more useful for the couple to buy the non-indexed lifetime annuity with a contingent survivor payout or whether on the contrary it is more attractive to acquire an indexed pension. Only in those cases where the acquisition percentages do not take on extreme values, 0% and 100%, can we obtain an order by which to compare them.

3.1.1.- Lifetime annuity with contingent survivor payout versus joint survivor annuity

If, through equivalent wealth, we compare the effect of buying a joint survivor annuity (JSA) instead of a lifetime annuity with contingent survivor payout on expected utility, we obtain the results in Table 3.

12 To determine this maximum percentage it has to be established that the wealth allocated to annuities is a control variable of the problem set. It should not be considered as taking on a fixed value equal to the initial wealth the individual has available at the time of retirement; this assumption is made to obtain the value of equivalent wealth. More details can be found in Lejárraga (2003).

Table 3

Equivalent wealth. Lifetime annuity with a 50% contingent survivor benefit (λ=0, φ=1). Comparison with a non-indexed joint survivor annuity.

δ↓	β→	β = 0.7		β = 2.9		β = 4.4	
		LA50%CSP	**JSA**	**LA50%CSP**	**JSA**	**LA50%CSP**	**JSA**
A) Non-indexed annuity.							
A		0.997	1.035	1.281	1.330	1.354	1.406
B		1.094	1.136	1.325	1.375	1.380	1.433
C		1.193	1.239	1.360	1.412	1.400	1.454
D		1.279	1.329	1.386	1.439	1.416	1.470
E		1.312	1.362	1.397	1.451	1.423	1.477
B) Indexed annuity.							
A		0.943	0.984	1.225	1.278	1.308	1.365
B		1.054	1.099	1.293	1.349	1.363	1.422
C		1.176	1.227	1.359	1.418	1.416	1.478
D		1.297	1.353	1.422	1.483	1.465	1.529
E		1.350	1.409	1.449	1.512	1.487	1.551

It can be seen that, with the purchase of joint survivor annuities, equivalent wealth follows the same pattern as in the case of annuities with a contingent survivor payout, according to the variation in the consumer's impatience and the degree of risk aversion he shows, as well as the relations between indexed and non-indexed pensions. Slightly greater utility is always achieved by buying joint survivor pensions. The results obtained in the following sections with regard to lifetime annuities with a contingent survivor benefit could therefore be extrapolated to joint survivor annuities.

3.1.2.-Sensitivity analysis

In all cases so far it has been considered that the degree of joint consumption, i.e. the percentage of consumption that can be shared, λ, is equal to 0, and that the weight of the woman's utility function with respect to the man's, φ, is equal to 1, which implies that both utility functions have the same weight. How do these specific values affect the resulting values for equivalent wealth?

When the degree of risk aversion is low, equivalent wealth is decreasing relative to the degree of joint consumption, and the greater the amount of consumption the couple can share, the lower the additional welfare achieved by buying a lifetime annuity with a contingent survivor benefit. However, if the degree of risk aversion is high, an increase in the value of λ provides an increase in equivalent wealth. Similar results can be seen in the paper by Brown & Poterba (2000).

Finally, it must be added that the values assumed for these variables do not appear to have a great deal of relevance in the analysis of equivalent wealth.

3.2.-Equivalent wealth and programmed withdrawal

In order to evaluate the results in the case where the couple allocate all their wealth to programmed withdrawal, we have considered that the annuity is bought by a man of 65 and that 50% will be payable to the surviving spouse, the latter being 62.

Table 4
Equivalent wealth for a couple allocating initial wealth to

δ↓	β→	i = 2%		i = 3%		i = 4%	
		$\beta = 0.7$	$\beta = 2.9$	$\beta = 0.7$	$\beta = 2.9$	$\beta = 0.7$	$\beta = 2.9$
A) A non-indexed pension under programmed withdrawal.							
A		0.802	0.892	0.903	1.000	1.008	1.109
C		0.885	0.892	0.994	1.000	1.104	1.109
E		0.892	0.892	1.000	1.000	1.109	1.109
B) An indexed pension under programmed withdrawal.							
A		0.747	0.881	0.853	1.000	0.964	1.122
C		0.856	0.881	0.974	1.000	1.096	1.122
E		0.881	0.881	1.000	1.000	1.122	1.122

The results obtained, shown in Table 4, are strongly influenced by two basic factors: the relation between the return achieved and the technical interest rate used in the calculation, and the degree of risk aversion.

If the return achieved on the account by the pension administrator (i) is lower than the interest rate anticipated (r), couples will prefer not to buy programmed withdrawal regardless of their impatience for consumption and their degree of risk aversion. In the case where the return achieved coincides with the rate of interest anticipated, programmed withdrawal will never be a clear preference. Finally, if a higher return than that generally anticipated is systematically achieved, the couple will prefer to use programmed withdrawal instead of having their resources invested in current wealth.

One aspect to highlight is the fact that for high risk aversion values, the equivalent wealth is constant, becoming independent of the couple's attitude towards consumption.

3.3.-Equivalent wealth and age on retirement[13]

A usual feature of public pension systems in different countries is the option the contributor has to take early retirement as long as he meets a number of specific requirements (date of entry to the system, years contributed, age, etc.). On the other hand, in countries where the system needs to be modified or transformed basically because of its potential medium or long-term inviability, pension reform generally includes the option to defer the age of retirement (normally voluntarily).

The consumer's age is a determining factor when choosing optimal consumption and deciding the best way of using the funds accumulated at the date of retirement. Early retirement brings about a decrease in equivalent wealth, or what amounts to the same thing, the welfare gain is more modest. On the other hand, deferring retirement increases the value the couple give lifetime annuities, given that the probabilities of dying are greater.

Just as happened in the case of the individual, the pattern followed by equivalent wealth under programmed withdrawal when bringing the retirement age forward or deferring it beyond

13 The results are not shown so as to avoid repetition. They are available on request from the authors.

what is considered normal varies according to the case observed. Under the first assumption (a very impatient consumer with little aversion to risk), the results are similar to those obtained when the couple allocate their wealth to the purchase of a lifetime annuity with a contingent survivor payout, i.e. equivalent wealth increases in line with retirement age. However, when the couple's risk aversion increases, the pattern depends on what the relation between the return achieved and the technical interest rate used in the calculation will be. With returns greater than the interest rate used, equivalent wealth becomes decreasing with retirement age. If the opposite is true, equivalent wealth becomes increasing. Nevertheless, the effect of age on the values calculated for wealth are small in both cases.

4. EQUIVALENT WEALTH FOR THE COUPLE AND MARKET *IMPERFECTIONS*

Generally speaking, couples with no bequest motives would obtain greater utility by buying a joint annuity with a contingent survivor benefit or a joint survivor life annuity through an insurance company than they would if they did not allocate their wealth to a pension, similarly to what occurred in the case of the individual. As is widely accepted, Blake (1999), annuities markets are not sufficiently well developed even in many of the more financially advanced countries, and so considering actuarially fair markets could therefore be thought too unreal an assumption.

What happens if so-called market *imperfections*[14] - mainly corrections for mortality and administration costs[15] - are included? Clearly the results for the utility achieved will vary considerably according to how the amount of the lifetime annuities is determined.

Along with mortality[16] rates and administration costs, another relevant factor in determining annuities is the technical interest rate used. In private pension systems the differences between the interest rate for the pension and the market interest rate are lessened when the contract includes profit sharing, which will be decided by the insurance company according to the surplus return obtained above the guaranteed technical interest rate on investments affecting the annuity policy. This participation will not normally be 100% of the surplus; the contract will establish a lower amount with a ceiling of 95%. The effect of this profit sharing for the pensioners will be an annual increase in the amount of the annuity.

In Table 5 we analyze what the results for equivalent wealth would be if we considered that the final value of the lifetime annuity with survivor payout obtained in equation (7) were redu-

14 Some authors do not agree at all with the term imperfection, and simply attribute the difference between the amount of pension in the market and an actuarially fair one to the price that has to be paid to the company for taking on the longevity risk.

15 Administration costs are an element to take into account in capitalization systems because, unlike what happens in pay-as-you-go systems, they become explicit. For more information on this point, see the paper by Devesa et al. (2002).

16 As Pierre Devolder has pointed out, the use of different mortality tables is not really the imperfection. If the tables used by the insurer give a fair estimate of the mortality rate of people choosing annuities (which is lower), there is no imperfection even though it means that tables other than those covering the general population are used. The imperfection arises from the "extra" safety margin the insurer can include, which is not justified judging from the experience of annuitants.

Table 5

Equivalent wealth. Lifetime annuity with a 50% contingent survivor benefit (λ=0, φ=1). Load factor 15%. Comparison with a non-indexed lifetime annuity (man).

δ↓	β→	β = 0.7		β = 2.9		β = 4.4	
		LA50%CSP	**Individual**	**LA50%CSP**	**Individual**	**LA50%CSP**	**Individual**
A) Non-indexed annuity.							
A		**0.847**	**0.928**	1.088	1.231	1.151	1.314
B		**0.930**	1.011	1.126	1.275	1.173	1.344
C		1.014	1.098	1.156	1.313	1.190	1.368
D		1.088	1.181	1.178	1.344	1.204	1.387
E		1.115	1.218	1.188	1.357	1.209	1.395
B) Indexed annuity.							
A		**0.801**	**0.892**	1.041	1.195	1.112	1.288
B		**0.896**	**0.985**	1.099	1.258	1.159	1.340
C		**0.999**	1.089	1.156	1.322	1.204	1.392
D		1.103	1.196	1.208	1.384	1.245	1.442
E		1.148	1.247	1.232	1.413	1.264	1.466

ced[17] by 15% (taking into account the combined effect of the different charges for the mortality rate and administration costs applied by the insurers and differences between the technical interest rate for the annuity and the rate offered on the market), using the same amount of wealth allocated to the purchase of a lifetime annuity. Although we use a load factor of 15 percent for all types of annuities, it should be noted that Finkelstein & Poterba (2004), using a unique data set consisting of annuities at a large UK company, find evidence that back-loaded annuities are priced higher, and annuities with payments to the estate are priced lower than other annuities.[18]

When the lifetime annuities available in the market are not "actuarially fair", some couples, specifically those with a low degree of risk aversion who are very impatient for consumption, achieve better welfare when they do not buy this type of pension, i.e. when they optimize their consumption path using the resources generated, at market interest rate, by the wealth accumulated at the date of retirement. However, a notable increase in welfare is still obtained in cases where attitude towards consumption could be considered more normal - levels C, D and E - although much lower than that achieved in the case of unmarried individuals.

Another relevant aspect is that equivalent wealth varies in all cases by the exact same percentage which determines the difference between what the couple would consider an "actuarially fair" joint annuity and that offered by the insurance company. In this case the equivalent wealth

17 Mitchell et al. (1999) calculate that for a 65-year-old male, the value of the annual pension is between 15 and 25 per cent less than would apply according to general mortality tables. Finkelstein & Poterba (2000) find that the value of life annuities bought voluntarily by men aged 65 in Great Britain is between 10 and 15 per cent less than they would be if the mortality tables for the population as a whole were used. Over 60% of this reduction is due to the longer life of pensioners when compared to the population average. James & Vittas (2000) study the life annuities markets in Canada, Great Britain, Switzerland, Australia, Israel, Chile and Singapore - a range of medium and high income countries - and find evidence of adverse selection, although with somewhat lower values than those of other authors cited for the same markets.

18 A more back-loaded annuity is one with a payment profile that provides a greater share of payments in later years.

Table 6

Maximum percentage of pension reduction for a lifetime annuity with a 50% contingent survivor benefit (λ=0, φ=1).

δ↓	β→	β = 0.7		β = 2.9		β = 4.4	
		LA50%CSP	Individual	LA50%CSP	Individual	LA50%CSP	Individual
A) Non-indexed annuity.							
A		**-0.3%**	8.4%	21.9%	30.9%	26.1%	35.3%
B		8.6%	15.9%	24.5%	33.3%	27.6%	36.7%
C		16.2%	22.6%	26.5%	35.3%	28.6%	37.9%
D		21.8%	28.1%	27.9%	36.8%	29.4%	38.7%
E		23.8%	30.2%	28.4%	37.4%	29.7%	39.1%
B) Indexed annuity.							
A		**-6.1%**	4.7%	18.4%	28.9%	23.6%	34.0%
B		5.1%	13.7%	22.6%	32.4%	26.6%	36.6%
C		15.0%	21.9%	26.4%	35.7%	29.4%	39.0%
D		22.9%	28.9%	29.7%	38.6%	31.7%	41.1%
E		25.9%	31.9%	31.0%	39.8%	32.8%	42.0%

needed to provide the same utility without buying annuities as allocating all the initial wealth available to a lifetime annuity is 15% lower than the wealth necessary if the pension purchased contained no charges or surcharges whatsoever on survival rates.

Table 6 show the calculations of the percentage by which the value of the lifetime annuity decreases over the value of the pension actuarially equivalent to the premium paid in the terms mentioned above - which the couple would be willing to accept given that it would still be more useful for them to buy the annuity than not do so.

As expected, all those cases where the maximum percentage admissible for reducing the pension is lower than 15% or negative show equivalent wealth values lower than one (in bold in Table 5). The most relevant thing about these two tables is that the couples place more value on lifetime annuities the less impatient they are to consume and the more risk aversion they show, just as in the case of individuals. Therefore, in the most extreme case analyzed, they would be willing to pay an insurance company up to 32.8% more than what would be actuarially fair in order to access an indexed lifetime annuity with a contingent survivor payout which would protect them against the risk of longevity. In all cases, the maximum percentage of reduction is lower (by between 7% and 10%) in the reversionary pension, which implies that buying annuities is less attractive for couples than for unmarried individuals, a fact already observed through the measurement of equivalent wealth.

5. EQUIVALENT WEALTH FOR COUPLES WITH PRE-EXISTING LIFETIME ANNUITIES WITH A CONTINGENT SURVIVOR BENEFIT

In some countries there is a certain amount of freedom of choice, and there may be pre-existing annuities proceeding from the first pillar of the pension system, as occurs in Argentina, Costa Rica and the United Kingdom, for example. This situation would include the case of couples in countries such as Spain, where there is a voluntary system which complements the public pension system (pay-as-you-go) and in which benefits are obtained solely in the form of lifeti-

Table 7

Equivalent wealth. Lifetime annuity with a 50% contingent survivor benefit (λ=0, φ=1), with 50% wealth in pre-existing annuities with survivor benefit.

δ↓	β→	β = 0.7		β = 2.9		β = 4.4	
		LA50%CSP	**Individual**	**LA50%CSP**	**Individual**	**LA50%CSP**	**Individual**
A) Non-indexed annuity.							
A		**0.920**	**0.959**	1.042	1.102	1.085	1.155
B		**0.975**	1.013	1.080	1.144	1.115	1.190
C		1.038	1.074	1.116	1.178	1.141	1.222
D		1.100	1.138	1.145	1.222	1.162	1.249
E		1.124	1.168	1.156	1.237	1.171	1.261
B) Indexed annuity.							
A		**0.895**	**0.939**	1.011	1.078	1.054	1.131
B		**0.955**	**0.998**	1.028	1.129	1.095	1.176
C		1.058	1.067	1.107	1.182	1.136	1.223
D		1.106	1.144	1.153	1.235	1.175	1.268
E		1.141	1.183	1.174	1.260	1.192	1.289

me annuities with a contingent survivor payout, and the option exists of transforming the accumulated savings into an additional lifetime annuity on retirement.

Table 7 shows the result for equivalent wealth for annuities with a 50% survivor payout (λ=0, φ=1), with 50% wealth in pre-existing annuities with survivor benefit. The most noticeable fact is that the equivalent wealth is lower than that obtained when there are no pre-existing lifetime annuities. In other words, when there are pre-existing lifetime annuities, the utility the couple achieve by buying additional lifetime annuities is much lower than it is if we assume they do not already have protection against the risk of longevity.

Thus, for example, for a couple with a risk aversion coefficient of 2.9, slightly impatient (D), and with 50% of their total wealth in an indexed lifetime annuity (see Table 7), converting the rest of their wealth into an indexed lifetime annuity brings them a 15.3% gain in welfare compared to the 42.2% gain the same couple would enjoy if all their wealth were to be converted into lifetime annuities (Table 1). In other words, a couple with a risk aversion coefficient of 2.9 and slightly impatient are willing to give up just 13.27% of their wealth in order to obtain a completely index-linked annuity, as against the 29.67% they would give up if they did not already have 50% of their wealth in a pre-existing annuity[19]. This is due to the pre-existing indexed annuity providing a fairly high minimum level of consumption.

Finally, in Table 8, using the same assumptions used to obtain the previous table and assuming that the couple already have half their initial wealth in a pre-existing lifetime annuity with 50% survivor payout and completely indexed against inflation, we calculate the percentage of the total initial wealth they would allocate to an additional annuity with survivor payout. As was to be expected through an analysis of the results in Table 7, allocating all the wealth to the pur-

19 Table 7: Beta=2.9 and slightly impatient: 1.153, therefore: 1-(1/1.153)=13.27%. Table 1: beta=2.9 and slightly impatient: 1.422, therefore: 1-(1/1.422)=29.67%.

Table 8

Equivalent wealth. Lifetime annuity with a 50% contingent survivor benefit (λ=0, φ=1), with 50% wealth in pre-existing annuities with survivor benefit.

δ↓	β→	Non-indexed annuity		Indexed annuity	
		$\beta = 0.7$	$\beta = 2.9$	$\beta = 0.7$	$\beta = 2.9$
A) A lifetime annuity with a 50% contingent survivor benefit which maximizes expected utility, with 50% wealth in pre-existing annuities with survivor benefit.					
A		0.00%	39.66%	0.00%	32.40%
C		43.12%	50.00%	37.67%	46.24%
E		50.00%	50.00%	50.00%	50.00%
B) A lifetime annuity with a 50% contingent survivor benefit which maximizes expected utility, with 50% wealth in pre-existing annuities with survivor benefit. Load factor 15%.					
A		0.00%	12.69%	0.00%	12.56%
C		0.00%	38.59%	0.00%	33.07%
E		50.00%	50.00%	50.00%	50.00%

chase of lifetime annuities does not turn out to be optimal in every case when 50% of the wealth is already invested in such instruments.

In conclusion it has to be said that the results described, along with the limitations of the model used, leave aside a number of presumably important aspects, such as:

a) The different tax treatment applied on the various types of benefit in different countries (which does not always work in favor of the life annuity option).

b) The non-consideration of bequest motives.

c) The exclusion of other possible dependents, which would have an effect on the results obtained in the case of couples.

d) The fact that life annuities are virtually the only product that can guarantee an interest rate in the long term.

This leads us to believe that the "annuity puzzle" is in fact no such thing since it is reasonably easy to explain why many individuals and couples avoid buying life annuities.

5.1.-Pre-existing lifetime annuities with survivor payout and market imperfections

If on top of there being pre-existing lifetime annuities we consider the fact that lifetime annuities bought in the market are not "actuarially fair", the results obtained reproduce the pattern seen in the case of single individuals.

Not only impatient or very impatient couples would be included within the category of those who would in theory prefer not to buy annuities. Also included would be couples with a noticeable degree of risk aversion and an attitude of indifference to consumption. In many cases the welfare gains obtained are less than 5%. The number of cases in which the couple would not choose to convert their wealth into an annuity is greater than for single men.

The results are completely confirmed by the percentage of total initial wealth that would be allocated to an additional annuity with survivor payout, the results for which are shown in Table 8. In some cases the couple would not allocate even a small part of their wealth to buy annuities. Only very slightly impatient couples would still allocate 100% of their wealth to buying an indexed lifetime annuity with survivor payout such as that described.

As can be seen from the results obtained, not only the consideration of market imperfections in the model but also the existence of previous annuities with survivor benefit strengthen the conclusion that the "annuities puzzle" as such does not exist.

If it were made compulsory to transform the savings accumulated in the individual capitalization account into a lifetime annuity, this could provide a solution to lessen the effects both of the myopia of individuals and moral hazard, both of which lead some people to consume their accumulated financial resources too quickly because they expect the government will provide them with the minimum necessary income. However, it is clear from the results that in some cases the couple may not find lifetime annuities an attractive option. For this reason the best thing would be for individual preferences and public policy objectives to be combined, thereby allowing wherever possible greater flexibility in choosing how the pension should be paid.

One way of balancing the two positions would be to establish that part of the accumulated fund should be used to purchase a life annuity that would provide the couple with a minimum periodic income related to previous earnings and/or the guaranteed minimum pension. This would eliminate the exposure of the State to the problem of the moral hazard of having to provide a pension to those retirees who use up their resources too quickly. Funds in excess of the amount needed to finance a sufficient level of income during retirement could be paid out in the form of a lump sum or programmed withdrawal. In those countries in which the retiree may be entitled to a pre-existing life annuity, this should be taken into account when calculating the minimum life annuity.

In voluntary defined contribution capitalization systems where the couple would presumably already have a large proportion of their wealth in the form of a life annuity, as would be the case of Spain, freedom of choice should be all the greater, without it being compulsory to earmark any money at all for the purchase of additional life annuities.

5.2.- Pre-existing lifetime annuities with survivor payout, market imperfections and age on retirement

In this final subsection we combine the effect of the pre-existing lifetime annuities with survivor payout and market imperfections with that produced by altering the age of retirement from the benchmark. As shown in Table 9, early retirement aggravates the effect described in the sense that it brings about an increase in the number of couples who would theoretically prefer not to buy the additional annuity and a decrease in equivalent wealth.

On the other hand, delaying retirement age provides the couple with greater welfare when they buy life annuities with survivor benefit than they obtain when they enter the annuities market at what is considered the normal age of retirement (age 65). This is because older people have a lower life expectancy and therefore obtain life annuities at a technical interest rate equivalent to or higher than the real market interest rate.

Table 9

Equivalent wealth according to retirement age. Indexed lifetime annuity with a 50% contingent survivor benefit (λ=0, φ=1), with 50% wealth in pre-existing annuities with survivor benefit. Load factor 15%.

(δ ; β)	Non-Indexed annuity			Indexed annuity		
	60	65	70	60	65	70
(A ; 0.7)	**0.840**	**0.852**	**0.867**	**0.816**	**0.831**	**0.848**
(C ; 0.7)	**0.943**	**0.953**	**0.966**	**0.934**	**0.944**	**0.958**
(E ; 0.7)	1.013	1.029	1.046	1.029	1.043	1.058
(A ; 2.9)	**0.937**	**0.952**	**0.973**	**0.907**	**0.926**	**0.949**
(C ; 2.9)	1.000	1.018	1.040	**0.992**	1.010	1.033
(E ; 2.9)	1.032	1.054	1.083	1.048	1.069	1.095

6.-CONCLUSIONS, POLICY RECOMMENDATIONS AND FUTURE DIRECTION

The results reached with the basic model confirm that welfare in the context of couples is lower than it is for individuals due to the fact that the couple's life expectancy is always greater than that of any two individuals taken separately. In general they do not differ from the results of other authors. In addition, some couples would prefer not to allocate all their wealth to buying a lifetime annuity, or what amounts to the same thing, they would obtain a greater welfare gain by being able to choose what percentage to allocate to the purchase of the lifetime annuity. Finally, because the differences between considering annuities with a contingent survivor payout and joint survivor lifetime annuities are very small, the results obtained with the former can be extrapolated to the latter.

With the aim of making it a closer reflection of the real situation in certain countries, some so-called market imperfections have been incorporated into the basic model. In this case, just as happened in the individual context, the non-availability of "actuarially fair" lifetime annuities on the market implies that couples with a small degree of risk aversion who are very impatient to consume achieve greater welfare by buying this type of annuity contract.

Factors other than market imperfections also exist that influence the decision to buy an annuity. The existence of previous lifetime annuities decreases the equivalent wealth obtained under the assumption that there are no previous lifetime annuities, and increases the number of cases in which the couple would prefer not to buy more lifetime annuities. Considering in addition that the lifetime annuities bought on the market are not "actuarially fair", still lower values of equivalent wealth are obtained, noticeably increasing the spectrum of couples that in theory would prefer not to buy additional lifetime annuities.

The models used in the analysis ignore certain presumably important questions -such as the different ways different types of pension are treated for tax purposes in some countries (which do not always benefit the option of receiving payment in the form of a lifetime annuity), the non-consideration of bequest motives, the fact that life annuities are virtually the only product that can guarantee an interest rate in the long term, and the exclusion of other possible dependents- which would accentuate the results obtained in the case of couples. This leads one to think that:

a) The so-called "annuity puzzle" is actually nothing of the kind, given that it is reasonably easy to explain why many individuals and couples resist buying life annuities.

b) The types of benefit available cannot be restricted exclusively to lifetime annuity contracts. It appears that it would be better for withdrawal regulations to have a certain amount of flexibility so as to accommodate individual circumstances and the aims of public policy. The most appropriate form seems to be that:

1. In mandatory defined contribution capitalization systems, a portion of the accumulated fund should be allocated to supply the couple with a minimum lifetime annuity related to the salary of the working person and/or to the guaranteed minimum pension. The exposure of the State to the obligation of paying a subsidy to those retirees who consume their resources too rapidly should be eliminated. Funds surplus to the amount needed to finance sufficient income during retirement could be paid out in a lump sum or via programmed withdrawal. In those countries where the retiree can count on a pre-existing lifetime annuity, this should be taken into account when calculating the minimum lifetime annuity.

2. Freedom of choice could be even greater in capitalization systems that complement defined benefit systems in which the couple presumably already have a large part of their wealth in the form of lifetime annuities, without the obligation to allocate any amount at all to the purchase of additional lifetime annuities.

Finally, further research could focus on:

a) Developing a model to include the bequest motive from the standpoint of couples.

b) Modeling the fact that life annuities are virtually the only product that can guarantee an interest rate in the long term. However, in the model followed in this paper all the other types of pension have been assigned a fixed return, which invalidates the supposed superiority of life annuities where this aspect is concerned. This could be very important when financial markets are undergoing periods of instability (which is almost always) and for those pensioners who are more averse to risk.

7. BIBLIOGRAPHY

— Bazaraa, M.S.; H.D. Sherali & C.M. Shetty (1993): "Nonlinear Programming, Theory and Algorithms". *John Wiley*. New York.

— Blake, D. (1999): "Annuity markets: Problems and solutions". *The Geneva Papers on Risk and Insurance* (24) 3, 358-375.

— Brown, J. R. (1999): "Private Pensions, Mortality Risk, and the decision to annuitize". *National Bureau of Economic Research*. WP-7191.

— Brown, J. R. (2000): "How should we insure longevity risk in Pensions and Social Security?". *Center for Retirement Research at Boston College*.

— Brown, J. R. (2001): "Private Pensions, Mortality Risk, and the decision to annuitize". *Journal of Public Economics*, 82 (1), 29-62.

— Brown, J. R & J.M. Poterba (2000): "Joint life annuities and annuity demand by Married Couples". *The Journal of Risk and Insurance 67* (4), 527-554.

— Brown, J., O. Mitchell & J.M. Poterba. (2001): "The Role of Real Annuities and Indexed Bonds in an Individual Accounts Retirement Program." In "Risk Aspects of Investment-Based Social Security Reform" Eds. Campbell, J and M Feldstein, *University of Chicago Press*, Chicago 2001, 321-370.

— Davidoff, T, J. Brown & P. Diamond (2003): "Annuities and Individual Welfare". *Center for Retirement Research on Pensions at Boston College*. WP 2003-11.

— Devesa, J. E., R. Rodríguez & C. Vidal (2002): "Assessing Administration Charges for the affiliate in Individual Accounts Systems". *IVIE. WP-EC* 2002-09.

— Devesa, J.E. & C. Vidal (2001): "Current Status and Provisional Assessment of Reformed Pensions Systems in Latin America" *The World Bank. World Bank Pension Reform Primer*.

— Frees, E., J. Carriere & E. Valdez (1996): "Annuity Valuation with Dependent Mortality" *Journal of Risk and Insurance*, 63 (2), 229-261.

— Finkelstein, A. & J. Poterba (2000): "Adverse Selection in Insurance Markets: Policyholder Evidence from The U.K. Annuity Market". *National Bureau of Economic Research*. WP-8045.

— Finkelstein, A. & J. Poterba (2004): "Adverse Selection in Insurance Markets: Policyholder Evidence from the U. K. Annuity Market.". *Journal of Political Economy*, 112 (1), 183-208.

— Friedman, B. & M.J. Warshawsky (1990): "The cost of annuities: implications for savings behavior and bequests". *Quarterly Journal of Economics*. 104 (2), 135-154.

— Halek, M. & Eisenhauer, J. G. (2001): "Demography of Risk Aversion" *Journal of Risk and Insurance, 68* (1), 1-24.

— James, E & D. Vittas (2000): "Annuities Markets in Comparative Perspective: Do Consumers Get Their Money's Worth?". *The World Bank. Policy Research*. WP-2493.

— Jianakoplos, N A. & A. Bernasek (1998): "Are Women more Risk Averse?". *Economic Inquiry 36* (4), 620-630.

— Kotlikoff, L.J. & A. Spivak (1981): "The family as an incomplete annuities market". *Journal of Political Economy 89*, 372-391.

— Lejarraga, A. (2003): "Modalidades de Pensión en los sistemas de Capitalización". *Doctoral Dissertation. University of Valencia*.

— Lejárraga, A., C. Vidal & J.E. Devesa (2002): "Regulación de las modalidades de pensión en los sistemas de capitalización de América Latina". *Revista de Análisis Económico*. 17- (2). 49-93.

— Mitchell, O. S.; J. M. Poterba; M. Warshawsky & J.R. Brown (1999): "New evidence on the money's worth of individual annuities". *American Economic Review 89* (5), 1299-1318.

— Powell, M. & D. Ansic (1997): "Gender differences in Risk Behaviour in Financial Decision-Making: An Experimental Analysis" *Journal of Economic Psychology,* 18 (6), 605-628.

— Valdés, S. & G. Edwards (1998): "Jubilación en los sistemas de pensiones privados". *El trimestre Económico Vol. LXV (1)*, Núm 257. 3-47.

— Valdés-Prieto, S. (2002): Políticas y mercados de pensiones. *Ediciones Universidad Católica de Chile, Santiago de Chile*.

— Ventura, M.; R. Meneu & J. Pérez-Salamero (2000): "Modelización y resolución de problemas de optimización en Economía". *Repro-Exprés, S.L.*, Valencia.

— Vidal, C; A Lejárraga; & J.E. Devesa (2002): "Regulating Withdrawals from Individual Pension Accounts in the Countries of Latin America and Decisions for the Consumer in Retirement." *5th Italian-Spanish Conference on Financial Mathematics*.

— Vidal, C & A Lejárraga (2004): "The Bequest Motive and Single People's Demand for Life Annuities". *Belgian Actuarial Bulletin*. 4 (2), 5-18.

— WorldBank (2001): "Annuities. Regulating withdrawals from individual pension accounts". *The World Bank. Social Protection Discussion Paper* N. 0107.

— Yaari, M. E. (1965): "Uncertain lifetime, life insurance and the Theory of the Consumer". *The Review of Economic Studies 32* (90), 137-150.

— Yagi, T. & Y. Nishigaki (1993): "The Inefficiency of Private Constant Annuities" *Journal of Risk and Insurance, 60* (3), 385-412.