Evaluating multi-beta pricing models: an empirical analysis with Spanish market data

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Abstract: Using Spanish stock market data running from January 1982 to December 1998, this paper examines competing models of price formation in security markets on the basis of the relationship between expected returns and different risk measures. The aim of the work is two-fold: to analyze whether the factor betas considered in each model have a significant role in explaining the behavior of average returns, and to compare the performance of the alternative models studied. We consider both static and conditional models, in which book-to-market and dividend yield aggregated ratios are chosen as predictors of changes in the market information set. We find that conditional models perform relatively better than static models.

Resumen: Sobre la base de la relación entre rentabilidad esperada y las diferentes medidas de riesgo que implican los distintos modelos de valoración de activos, el objetivo de este trabajo consiste en examinar el comportamiento de diferentes modelos de formación de precios, usando datos del mercado español bursátil en el periodo comprendido entre enero de 1982 y diciembre de 1998. Este objetivo se pretende alcanzar desde dos tipos de análisis: por un lado, comprobando si las betas asociadas a los factores de riesgo considerados por cada modelo son relevantes en la explicación de los rendimientos medios, y por otro, comparando el ajuste de cada modelo alternativo a los datos de este estudio. Consideramos tanto modelos con carácter estático como modelos que incorporan dinamismo al estar especificados en términos condicionales, en los que las variables utilizadas como predictoras de los cambios en el conjunto de información de la economía son el ratio valor contable/valor de mercado y la rentabilidad por dividendos, ambos agregados. Los resultados muestran que los modelos condicionales presentan un mejor comportamiento que los modelos estáticos.

JEL classification: E44, G12.

Keywords: Stock markets, factor models, risk prices, performance.

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1. INTRODUCTION

Quantifying the relationship between expected return and risk is a basic issue in asset pricing theory. The Capital Asset Pricing Model (CAPM), (Sharpe (1964) and Lintner (1965)); the Arbitrage Pricing Theory (APT) family with alternative approaches based on macroeconomic variables (Chen et al. (1986)), statistical factors (Connor and Korajczyk (1986, 1988)), or firm characteristics (Fama and French (1992, 1993)); intertemporal models using explicit and dynamic hedging behavior among investors (Merton (1973) and Campbell (1993)); the conditional CAPM (Jagannathan and Wang (1996)); and nonlinear pricing kernel models (Dittmar(2002)) are the key asset pricing models dominating empirical and theoretical literature in Financial Economics.

This work provides a detailed comparative analysis of the empirical performance of five multi-beta asset pricing models from among those mentioned above. For traditional reasons, the standard CAPM is also considered in the analysis. In our estimations, monthly returns of Spanish stocks from 1982 to 1998 are employed. Using the well known Fama and MacBeth (1973) two-step regressions, we study the explanatory power of the risk factors associated with each model by estimating their risk premiums. In addition, we also directly compare the models, employing the Hansen and Jagannathan (1997) distance.

Surprisingly, despite the enormous amount of empirical literature available, there are very few papers which directly compare asset pricing models using either the procedure suggested by Hansen and Jagannathan or alternative approaches specifically designed to compare pricing models on a fair basis. Important exceptions are Jagannathan and Wang (1996), Jagannathan et al. (1998), Brennan et al. (1998), and Ferson and Harvey (1999). This paper presents further international evidence along these lines, by employing data from a thinly traded and order driven securities market. In 1989, the Spanish market became a continuous auction market in which execution against limit orders left on the computerized public book is allowed by the trading mechanism. As usual in these types of market, by monitoring available bids and offers on the book, stock exchange agencies (brokers) can execute upcoming orders against an existing bid or offer. Alternatively, of course, they can introduce a new sale or purchase order. Recent cross-sectional evidence on the determinants of average stock returns in markets characterized by the mechanism briefly described above is very limited.

It should be pointed out that our comparative analysis includes a cross-sectional version of the intertemporal asset pricing model proposed by Campbell (1993). To the best of our knowledge, this is the first time that such a version has competed directly with the more traditional pricing models. Finally, the aggregate book-to-market ratio is used as a predictive factor in the conditional CAPM. Again, this is the first time that such a factor has been used in that context. As it turns out, both the intertemporal model of Campbell and the conditional CAPM perform relatively better than the static models considered: CAPM, APT models, and an extension of the CAPM using a cubic pricing kernel in the market return.

1 This is reasonable given the evidence reported by Kothari and Shanken (1997), Lewellen (1999), and Pontiff and Schall (1999) of the power of an aggregate book-to-market ratio in predicting returns.
The rest of the paper is organized as follows: Section 2 reports the data used in this research. In section 3, we briefly describe the models analyzed in the empirical comparison. The models are cross-sectionally estimated in section 4. In section 5, the models are estimated again by the Generalized Method of Moments (GMM). The target is now, of course, to compare the models using the distance metric proposed by Hansen and Jagannathan (1997). Finally, in section 6 we draw our conclusions.

2. DESCRIPTION OF THE DATA

Monthly returns from January 1982 through December 1998 are employed in this research. This gives a total of 204 observations. Our objective is to explain the average returns of ten portfolios constructed by sorting 167 stocks into size deciles based on their market value at the end of the previous year. The returns of each portfolio are equally-weighted. To approximate the return on total wealth we use the return on an equally-weighted portfolio comprised of all stocks available in a given month, or the return on the usual value-weighted index. The monthly equivalent of the one-year Treasury Bill rate observed in the secondary market has been used as the risk-free rate.

Three additional variables have also been used to construct either explanatory variables or risk factors in the different models. These are: a size proxy, the book-to-market aggregate ratio (BM), and the dividend yield aggregate ratio (DY). As a measure of size for each company in a single month we use the logarithm of market capitalization, calculated by multiplying the number of shares of each firm in December of the previous year by their price at the end of each month. An aggregate measurement of this variable is calculated as the equally-weighted average of all market values available in a particular month. To compute the book-to-market ratio for each firm, we employ the accounting information from the balance sheets of each firm at the end of each year. Since 1990, this information has been provided by the National Security Exchange Commission. Data for the years before 1990 is obtained from the quarterly bulletins published by the Stock Exchange Market. The book value for any firm in month t is given by its value at the end of the previous year, and it remains constant from January to December. The market value is given by total capitalization of each company in the previous month. The corresponding aggregate BM is computed as the average of the individual BM ratios. The dividend yield, as an aggregate variable, is obtained as the arithmetical average of the dividend yields of each firm in the sample. The individual DY for a given month, is computed as the total dividends paid by the firm during the previous twelve months divided by the price at the end of the last month.

3. COMPETING ASSET PRICING MODELS

We can summarize the asset pricing models with the following equation,

\[ E\left[ \tilde{R}_t M_t / \Omega_{t-1} \right] = 1 \]

where \( E \) denotes expectation; \( \tilde{R}_t \) is the gross return on asset i at time t; \( M_t \) is the stochastic discount factor (SDF); and \( \Omega_{t-1} \) is the information set available at time t-1.
This is the fundamental equation for all the models we consider. It is known as the SDF representation. To write each one we only have to specify the SDF as a function of given factors.

Another commonly used representation for factor models consists of expressing the above equation as a linear relationship between expected returns and systematic risk measures. This is the beta representation, which is derived below.

Using the definition of covariance in (1),

$$ E \left( \tilde{R}_i / \Omega_{t-1} \right) E \left( M_t / \Omega_{t-1} \right) + \text{Cov} \left( R_g, M_t / \Omega_{t-1} \right) = 1 $$

where $R_g = \tilde{R}_g - 1$, and solving for the conditional expected return of asset $i$:

$$ E \left( \tilde{R}_i / \Omega_{t-1} \right) = \frac{1}{E \left( M_t / \Omega_{t-1} \right)} - \frac{\text{Cov} \left( R_g, M_t / \Omega_{t-1} \right)}{E \left( M_t / \Omega_{t-1} \right)} $$

(3)

Let $\tilde{R}_u$ be the gross return on a portfolio with zero covariance with respect to the SDF. From (1), we can see that its conditional expected net return ($\gamma_{0t-1}$) is given by

$$ \gamma_{0t-1} + 1 = E \left[ \tilde{R}_0 / \Omega_{t-1} \right] = \frac{1}{E \left( M_t / \Omega_{t-1} \right)} $$

(4)

Let $\gamma_{t-1}$ be the risk premium conditional on the information available at $t-1$ and let $\beta_{g-1}$ be the measure of conditional systematic risk of asset $i$, that is,

$$ \gamma_{t-1} = -\frac{\text{Var} \left( M_t / \Omega_{t-1} \right)}{E \left( M_t / \Omega_{t-1} \right)} $$

$$ \beta_{g-1} = \frac{\text{Cov} \left( R_g, M_t / \Omega_{t-1} \right)}{\text{Var} \left( M_t / \Omega_{t-1} \right)} $$

we can write (3) as a linear function between the expected conditional return on an asset and its conditional systematic risk:

$$ E \left( \tilde{R}_i / \Omega_{t-1} \right) = \gamma_{0t-1} + \gamma_{t-1} \beta_{g-1} $$

(5)

If we assume the existence of a risk free asset the above equation still holds with $\gamma_{0t-1}$ replaced by $R_f$, where this last variable denotes the risk free net return.

We specify the equations above to obtain six representative models of asset pricing literature, which will be estimated in the following section. In describing them, we argue that it is appropriate to distinguish between equilibrium and arbitrage based asset pricing models.
3.1. Models based on equilibrium conditions

In an equilibrium framework, equation (1) arises as the solution to the intertemporal consumption and portfolio choice problem of the representative agent, and the SDF represents the intertemporal marginal rate of substitution of aggregate consumption.

\[ M_t = \frac{U'(C_t)}{U'(C_{t-1})} \]  

(6)

where \( U' \) denotes marginal utility, \( C_t \) aggregate consumption at time \( t \), and the subjective rate of time preference is implicit in the utility function.

**Capital Asset Pricing Model (CAPM)**

The first model that we consider is the standard CAPM. Its static character is its most relevant characteristic. It means that each period of time is independent from the rest, which allows us to replace aggregate consumption in the utility function by aggregate wealth (\( W_t \)).

\[ M_t = \frac{U'(W_t)}{U'(W_{t-1})} \]  

(7)

Making some assumptions about the utility function\(^2\) or assuming normality in the distribution of returns, the stochastic discount factor can be written as a linear function of the return on wealth (\( R_{mt} \)).

\[ M_t = \delta_0 + \delta_m R_{mt} \]  

(8)

So, using (1) and taking into account that the unconditional and conditional moments are the same in this context, the SDF representation of this model is

\[ E\left[ \tilde{R}_u (\delta_0 + \delta_m R_{mt}) \right] = 1 \]  

(9)

And its beta representation

\[ E(R_u) = \gamma_0 + \gamma_m \beta \]  

(10)

with \( \beta = Cov(R_u, R_{mt})/Var(R_{mt}) \), where the relationship between the parameters in (10) and (9) is\(^3\):

\[ \gamma_0 = \frac{1}{\delta_0 + \delta_m Var(R_{mt})} - 1 \quad \gamma_m = -\delta_m (\gamma_0 + 1)Var(R_{mt}) \]  

(11)

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\(^2\) The assumption of quadratic preferences is the most common way to obtain this model.

\(^3\) The relationship between the parameters in both SDF and beta representations in all the models presented here can be obtained in the same way.
Conditional CAPM

The static assumption is the most important limitation in the intuitive simple CAPM. Since expected returns and betas depend on the available information at each moment in time, it seems reasonable to modify the traditional static model to incorporate time-varying expected returns. Jagannathan and Wang (1996) suggest a conditional version of the static CAPM by simply considering changes in public information available to market participants. Thus, the beta representation of the model has the form of equation (5),

\[
E(R_{it}/\Omega_{t-1}) = \gamma_0 + \gamma_{mt-1} \beta_{it-1}
\]  

(12)

where the conditional systematic risk is now given by

\[
\beta_{it-1} = \frac{\text{Cov}(R_{it}, R_{mt}/\Omega_{t-1})}{\text{Var}(R_{mt}/\Omega_{t-1})}
\]

(13)

Taking unconditional expectations on both sides of (12), we can write the unconditional expected return on any asset as a linear function of its expected beta, and its beta-premium sensitivity, so that we have an unconditional model with two factors.

\[
E(R_{it}) = \gamma_0 + \gamma_m E(\beta_{it-1}) + \text{Cov}(\gamma_{mt-1} \beta_{it-1})
\]

(14)

Under this framework, the investor expects to obtain a higher return on those assets not only with higher beta risk, but also with higher covariance between beta risk and expected risk premium.

To empirically implement the model, it is necessary to make some assumptions:

1) Following Jagannathan and Wang (1996), we use the unconditional beta as a proxy for the expected conditional beta and the sensitivity of the return on assets to changes in the conditional risk premium as the proxy for the covariance in (14).

\[
E(\beta_{it-1}) = \beta_i, \quad \text{Cov}(\gamma_{mt-1}, \beta_{it-1}) = \text{Cov}(\gamma_{mt-1}, R_{it})
\]

(15)

2) Since the market risk premium varies throughout the business cycle we choose the aggregate \(DY\) as well as the aggregate \(BM\) ratios as state variables in the information set \(\Omega_{t-1}\), given their forecasting power in the Spanish market between 1982 and 1998 (Nieto, 2002), and we assume the following linear relationship:

\[
\gamma_{mt-1} = \kappa_0 + \kappa_1 BM_{t-4} + \kappa_2 DY_{t-4}
\]

(16)

Combining (15) and (16) with (14), the model to be tested is therefore given by:

\[
E(R_{it}) = \gamma_0 + \gamma_m \beta_i + \gamma_{bm} \beta^{bm}_i + \gamma_{dy} \beta^{dy}_i
\]

(17)
where $\beta_{it}^{bm}$ and $\beta_{it}^{dy}$ are obtained by regressing the returns of each asset on the lagged values of both the BM and DY ratios.

Then, the SDF representation of this model is:

$$E \left[ \tilde{R}_{it} (\delta_0 + \delta_m R_{mt} + \delta_{bm} BM_{t-1} + \delta_{dy} DY_{t-1}) \right] = 1$$

\[ (18) \]

\[ \text{Inter temporal Model without Consumption} \]

It should not be forgotten that in a dynamic context the utility function depends on aggregate consumption and the beta representation of a pricing model includes the covariance between return on assets and changes in consumption. The main problem in this type of models is the difficulty of measuring aggregate consumption appropriately. The goal of the framework proposed by Campbell (1993) is to obtain an intertemporal model which does not need consumption data.

First, Campbell obtains a multi-factor model from the utility function of the representative agent suggested by Epstein and Zin (1989) and Weil (1989) with recursive-non-separable preferences, by maximizing the expected utility of consumption subject to an intertemporal budget restriction in which the relationship between consumption and wealth is used to substitute consumption from the first order condition. Additionally, joint conditional log-normality of asset returns and consumption is assumed.

Specifically, from the budget constraint of the representative agent, we may write the innovations in consumption as a function of innovations in the return on wealth and the revision of expectations of the future return on wealth. This allows us to substitute the covariance between returns and changes in consumption for the covariance between the returns and the innovations on both the return on wealth and state variables with ability to forecast returns.

If we again consider that BM and DY aggregate ratios are appropriate state variables, the revision in expectations of future return on wealth would be reflected in error terms of the following auto-regressive vector (VAR):

$$Z_t = AZ_{t-1} + \epsilon_t$$

\[ (19) \]

with $Z_t = (R_{mt}, BM_t, DY_t)$, $\epsilon_t = (\epsilon_{mt}, \epsilon_{bm}, \epsilon_{dy})$, and $A$ being the parameter matrix.

Assuming the existence of a risk free rate, the beta specification of the model would be:

$$E_t (R_{it}) - R_{ft} = \gamma_m \beta_{ti}^{m} + \gamma_{bm} \beta_{ti}^{bm} + \gamma_{dy} \beta_{ti}^{dy}$$

\[ (20) \]

where $\beta_{ti}^{m} = \frac{\text{Cov}(R_{it}, \epsilon_{mt})}{\text{Var}(\epsilon_{mt})}$, $\beta_{ti}^{bm} = \frac{\text{Cov}(R_{it}, \epsilon_{bm})}{\text{Var}(\epsilon_{bm})}$, and $\beta_{ti}^{dy} = \frac{\text{Cov}(R_{it}, \epsilon_{dy})}{\text{Var}(\epsilon_{dy})}$. 
It should be noted that the $BM$ and $DY$ ratios play different roles in the conditional CAPM and in this intertemporal asset pricing model. In the model described by equation (20), betas arise from the substitution of aggregate consumption by alternative state variables that contain information on future returns. Hence, the explanatory variables of the model are not covariances between the return of the assets and the (lagged) factors, but covariances between the returns and innovations in the factors (the error terms from the VAR system).

Finally, the Campbell model admits the following SDF representation:

$$E \left[ \tilde{R}_t \left( \delta_0 + \delta_m \epsilon^m_t + \delta_{bm} \epsilon^{bm}_t + \delta_{dy} \epsilon^{dy}_t \right) \right] = 1$$  \hspace{1cm} (21)

Since the factors in this model are the innovations in the variables with the ability to forecast future returns, and they follow a first-order VAR, we can substitute them in the moment restriction:

$$E \left[ \tilde{R}_t \left( \theta_0 + \delta_m R_{mt} + \delta_{bm} BM_t + \delta_{dy} DY_t + \theta_1 R_{mt-1} + \theta_2 BM_{t-1} + \theta_3 DY_{t-1} \right) \right] = 1$$  \hspace{1cm} (22)

where $\theta$ s are a combination of the VAR parameters and $\delta$ s.

**Nonlinear pricing kernel**

In all the models above, the stochastic discount factor is expressed as a linear function of the factors that are assumed to be the explanatory variables for average returns. For this to be done, either a utility function or a return distribution must be assumed. However, there is no known suitable representation for the utility function and empirical studies of nonparametric models, such as Bansal et al. (1993), Bansal and Viswanathan (1993), and Chapman (1997), seem to explain the cross-sectional variation in expected returns better than, for example, the CAPM, suggesting nonlinearities in the data. The model presented here seeks to address these issues.

First, as with the CAPM, we assume a static setting where consumption and wealth are equivalent, and the intertemporal rate of substitution can be expressed as a function of aggregate wealth (equation (7)). To avoid making assumptions about the form of the utility function we can approximate the SDF using a Taylor expansion around the return on aggregate wealth:

$$M_t = h_0 + h_1 \frac{U''}{U'} R_{mt} + h_2 \frac{U'''}{U''} R_{mt}^2 + ...$$  \hspace{1cm} (23)

Second, to pick up the nonlinearities observed in data, we must consider a Taylor expansion of order bigger than one and, of course, we have to decide on the point of truncation$^4$. As in the nonparametric analysis mentioned before, we could allow data to guide us in this issue; however, the problem with this approach is a loss of power due to overfitting. Alternatively, we may use preference theory. In this way, following the work of Dittmar (2002), we take into account the first three polynomial terms in the Taylor expansion.

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$^4$ If we only use the standard arguments of positive marginal utility and risk aversion the expansion should be truncated after the linear term, and this approximation is consistent with the static CAPM.
Dittmar argues that the choice of this point of truncation is reasonable because it is possible to sign these first three terms using risk aversion theory. Positive marginal utility \((U' < 0)\) and risk aversion \((U'' < 0)\) imply a negative coefficient for the return on aggregate wealth. Decreasing absolute risk aversion \((U'' > 0)\), consistent with the findings of Arditti (1967), implies a positive coefficient on the quadratic term. Finally, Kimball (1993) shows that decreasing absolute prudence is needed too \((U''' < 0)\), so that the coefficient on the cubic term is restricted to being negative.  

Therefore, we investigate the pricing fit of a model with an SDF of the following form,

\[
M_t \approx h_0 + h_1 \frac{U''}{U'} R_{mt} + h_2 \frac{U'''}{U'} R_{mt}^2 + h_3 \frac{U''''}{U'} R_{mt}^3
\]  

(24)

where \(\delta_1\), in order to be consistent with the restrictions implied by preference theory, must be negative, \(\delta_2\) positive, and \(\delta_3\) negative.  

Its SDF and beta representations are given respectively by:

\[
E\left[ \tilde{R}_t \left( \delta_0 + \delta_1 R_{mt} + \delta_2 R_{mt}^2 + \delta_3 R_{mt}^3 \right) \right] = 1
\]  

(26)

\[
E(R_{it}) = \gamma_0 + \gamma_1 \beta_{im1} + \gamma_2 \beta_{im2} + \gamma_3 \beta_{im3}
\]  

(27)

where

\[
\beta_{imk} = \frac{\text{Cov}(R_{it}, R_{km})}{\text{Var}(R_{it})}, \quad k=1,2,3.
\]

3.2. Models based on Arbitrage Pricing Theory (APT)

Another kind of factor pricing model is obtained using absence of arbitrage opportunities instead of equilibrium arguments. In this case, it is only necessary to assume a determined generating process for asset returns and to use the mimicking portfolio concept. The idea relies on the Law of One Price: if an asset has the same systematic risk as the portfolio (same payoff in each state of nature), both must have the same expected return.

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5 Samuelson (1963) proves that if an agent had already accepted a bet with a negative expected payoff, he should be unwilling to accept another independent bet with a negative expected payoff too. Thus, Kimball (1993) maintains that the standard risk aversion behaviour needs decreasing absolute prudence,

\[
- \frac{dU'''}{dW} = \frac{(U''')^2 - U''''U''}{(U'')^2} < 0
\]

in addition to decreasing absolute risk aversion, implying \(U''' < 0\).

6 In the estimation of coefficients of (26) we restrict the signs to agree with preference theory.
Consider the following multi-factor generating process of asset returns:

\[ R_{it} = E(R_{it}) + \sum_{k=1}^{K} \beta_{ik} F_{kt} + e_{it}, \quad i = 1, 2, \ldots, N \]  

(28)

where \( \beta_{ik} \) are the innovations in the factors of systematic risk. The exact version of the APT implies that the expected return on assets is a linear and positive function of the betas related to the risk factors. If we assume the existence of a risk-free rate the equation is,

\[ E(R_{it}) = R_{ft} + \sum_{k=1}^{K} \gamma_{kt} \beta_{ik} \]  

(29)

where \( \gamma_{kt} \) are the expected risk premiums of the factors.

Combining the equations (28) and (29) we obtain the appropriate form for testing the restrictions implied by the APT:

\[ R_{it} - R_{ft} = \sum_{k=1}^{K} F_{kt} \beta_{ik} + e_{it} \]  

(30)

where \( F_{kt} = \gamma_{kt} + \gamma_{kt} \beta_{ik} \) is the risk premium carried by the (unobservable) factor k.

**The Three-Factor Model of Fama and French**

In the search for factors that mimic the systematic risk of assets, Fama and French (1993) propose a model in which the expected returns on each asset are related to three “risk factors” suggested by previous cross-sectional evidence on asset pricing\(^7\). The excess return of the market portfolio over the risk-free rate, and two mimicking portfolios for size and book-to-market risk factors are included in the model. The theoretical relationship between expected returns and their risk factor betas, if in fact their mimicking portfolios are truly aggregate risk factors, should be given by:

\[ E(R_{it}) - R_{ft} = \beta_{im}^{m} E(R_{mt} - R_{ft}) + \beta_{i}^{SMB} E(SMB_{i}) + \beta_{i}^{HML} E(HML_{i}) \]  

(31)

where \( SMB_{i} \) represents the (associated) size factor obtained as the difference between the return on small and big assets with about the same weighted-average \( BM \), \( HML_{i} \) is the (associated) \( BM \) risk factor calculated as the difference between the return on assets with high and low book-to-market ratios (also controlling for size), and betas are the sensitivities of the returns to the corresponding risk factors.\(^8\)

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\(^7\) Authors such as Banz (1981) find a size effect in the returns, Basu (1983) and Rosenberg et al. (1985) show the existence of a relationship between average returns and financial ratios such earnings-to-price or book-to-market equity.

\(^8\) See Fama and French (1993) for details of the construction of the factors.
Lastly, we consider a general APT model in which factors are unknown. To estimate the unobservable factors in (30), we employ the asymptotic principal components suggested by Connor and Korajczyk (1986, 1988). Assume that we observe returns on N risky assets and the risk-free rate over T time periods. Let \( r^N \) denote the NxT matrix of observed returns on N risky assets in excess of the risk-free rate, \( r^N = R^N \cdot R^{-} \cdot R^{T} \). Let \( B^N \) denote the NxK matrix of factor sensitivities; \( F \) is the KxT matrix of realizations of \( (g + F) \), and \( e^N \) is the NxT matrix of realized idiosyncratic returns. Equation (30) can be written as:

\[
E\left[ \bar{R}^N \left( \delta_0 + \delta_1 R_{m,t} + \delta_2 SMB_t + \delta_3 HML_t \right) \right] = 1
\]  

(32)

Asymptotic Principal Components as Risk Factors

Our objective is to estimate the unobservable values of \( F \) given observations of \( r^N \) and an assumed value of \( K \). It can be shown that the factors can be (asymptotically as \( N \) grows larger) approximated by the first \( K \) eigenvectors of the TxT matrix \( \Omega^N = (1/N) r^N r^N \).

As we will see in the next section, we identify three relevant factors, which are denoted by \( f_k \), \( k = 1,2,3 \). So the model to be tested in terms of betas is:

\[
E\left( R^N \right) - R^N = \gamma \beta_{i1} + \gamma \beta_{i2} + \gamma \beta_{i3}
\]  

(34)

where \( \beta_{ia} = \frac{Cov(R^N, f_{iN})}{Var(f_{iN})} \), \( k=1,2,3 \)

And the SDF representation for this APT model is:

\[
E\left[ \bar{R}^N \left( \delta_0 + \delta_{1} f_{1t} + \delta_{2} f_{2t} + \delta_{3} f_{3t} \right) \right] = 1
\]  

(35)

4. CROSS-SECTIONAL ESTIMATION OF RISK PREMIUMS IN BETA REPRESENTATIONS

4.1. Some preliminary evidence

In Table 1, we first summarize the principal descriptive statistics of the ten size-sorted portfolios used as dependent variables as well as the risk factors of the Fama-French model. The mean return of the portfolios decreases as their size increases. In fact, small firms earn on average 1% per month more than large firms. A similar finding is observed regarding volatility, although the standard deviations of portfolios 3 to 9 are approximately the same. In the second panel of Table 1 we report statistics referring to risk factors. The market return is the return on the equally-weighted market index. The size and book-to-market associated factors offer a similar mean return and, at the same time, a much smaller mean and standard deviation than the market return. None of the three factors seems to have large autocorrelations and, as expected by
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### TABLE 3.
Summary statistics for factors from asymptotic principal components

<table>
<thead>
<tr>
<th></th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.012059</td>
<td>0.011085</td>
<td>-0.006520</td>
</tr>
<tr>
<td>Median</td>
<td>0.008932</td>
<td>0.005282</td>
<td>-0.004215</td>
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<tr>
<td>Maximum</td>
<td>0.257268</td>
<td>0.260980</td>
<td>0.284368</td>
</tr>
<tr>
<td>Minimum</td>
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<td>-0.209835</td>
<td>-0.405335</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.069137</td>
<td>0.069301</td>
<td>0.069881</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.094073</td>
<td>0.067161</td>
<td>-0.418575</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.275111</td>
<td>3.974150</td>
<td>10.48124</td>
</tr>
</tbody>
</table>

Correlations

<table>
<thead>
<tr>
<th></th>
<th>$R_m$ equally-weight.</th>
<th>$R_m$ value-weight.</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>0.9860</td>
<td>0.8537</td>
<td>1</td>
<td>0.0280</td>
<td>-0.0164</td>
</tr>
<tr>
<td>$f_2$</td>
<td>0.0280</td>
<td>0.1389</td>
<td>1</td>
<td>1</td>
<td>-0.0150</td>
</tr>
<tr>
<td>$f_3$</td>
<td>-0.0164</td>
<td>-0.0150</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

This table is based on 204 monthly observations (1982:1-1998:12). $f_k$’s mimic risk factors and they are estimated by the asymptotic principal components technique proposed by Connor and Korajczyk (1988). All stocks for which data are available throughout the sample period are employed.

### TABLE 4.
OLS estimates from $R_m - R_p = \alpha + \beta_1 f_1 + \beta_2 f_2 + \beta_3 f_3 + \epsilon_t$

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$R^2$</th>
<th>$R^2$ adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A:</strong> equally-weighted index</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 factor</td>
<td>0.001297</td>
<td>1.061152*</td>
<td></td>
<td></td>
<td>0.9720</td>
<td>0.9719</td>
</tr>
<tr>
<td></td>
<td>1.463</td>
<td>83.778</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 factors</td>
<td>0.000625</td>
<td>1.062807*</td>
<td>0.058895*</td>
<td></td>
<td>0.9750</td>
<td>0.9748</td>
</tr>
<tr>
<td></td>
<td>0.734</td>
<td>88.561</td>
<td>4.919</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 factors</td>
<td>0.000589</td>
<td>1.062894*</td>
<td>0.058795*</td>
<td>-0.005144</td>
<td>0.9751</td>
<td>0.9747</td>
</tr>
<tr>
<td></td>
<td>0.688</td>
<td>88.376</td>
<td>4.915</td>
<td>-0.432</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel B:</strong> value-weighted index</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 factor</td>
<td>0.003052</td>
<td>0.844810*</td>
<td></td>
<td></td>
<td>0.7300</td>
<td>0.7287</td>
</tr>
<tr>
<td></td>
<td>1.206</td>
<td>23.371</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 factors</td>
<td>0.001215</td>
<td>0.849329*</td>
<td>0.160823*</td>
<td></td>
<td>0.7566</td>
<td>0.7542</td>
</tr>
<tr>
<td></td>
<td>0.498</td>
<td>24.674</td>
<td>4.683</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 factors</td>
<td>0.000456</td>
<td>0.851195*</td>
<td>0.162538*</td>
<td>-0.109991*</td>
<td>0.7692</td>
<td>0.7658</td>
</tr>
<tr>
<td></td>
<td>0.190</td>
<td>25.329</td>
<td>4.848</td>
<td>-3.309</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results are based on 204 monthly observations (1982:1 - 1998:12). $f_k$’s mimic risk factors and are estimated by the asymptotic principal components technique proposed by Connor and Korajczyk (1988). All stocks for which data are available throughout the sample period are employed. $R_m$ is the monthly return on an equally-weighted index (panel A) or value-weighted index (panel B) constructed with all stocks in the sample. $t$-values are under the estimations. * indicates significance at 5% level and ** at 10%.
employed in estimation. Then, we obtain the eigenvectors of this matrix, and choose the three eigenvectors associated with the higher eigenvalues. These will be taken as proxies of the factors in our APT model specification.

In Table 3, some descriptive statistics of these factor proxies are reported. Their means are relatively small because the observations are in a range between -25% and 25% approximately. This results in a (monthly) standard deviation of 7%, which is similar to the one reported for our two market indexes. It is interesting to point out that there is a correlation of more than 98% between the first factor and the equally-weighted index, and of 85% with the value-weighted. This suggests that this factor adequately captures the variability of assets in our sample. To corroborate, a regression of the excess market return on these three estimated factors is performed. Although a high correlation between the market and the factors is not enough to guarantee that these are reasonable proxies for the factors, a low $R^2$ will indeed indicate a lack of suitable factors.

In Table 4, we show the results of the estimated slopes and the $R^2$-squares, for both an equally-weighted index (panel A), and a value-weighted index (panel B). There is a very small improvement in explaining the variability of market returns as we add more than one factor to the regressions. It should be noted that the first factor explains more than 97% of the variance of the equally-weighted index. Although the remaining factors have some explanatory power, it seems that they explain a much lower proportion of the variance. Another interesting feature is that the $R^2$-square is consistently higher when using the equally-weighted portfolio instead of the value-weighted index. The improvement in the explanatory capacity achieved by increasing the number of factors from one to three is marginal, although it is clearly greater for the value-weighted market return. These results are certainly encouraging. Given that the methodology employed is based on asymptotic results, and taking into account the small number of securities used in the cross-section, lack of correlation might have indicated that our estimated factors omit relevant state variables by the market. But the results do indeed seem to support our estimates as reasonable APT factors.9

4.2. Fama and MacBeth estimation

We now perform the traditional two-step cross-sectional regressions due to Fama and MacBeth (1973). The objective is to test whether the risk premium associated with each factor is statistically positive and significant.

The average Ordinary Least Square (OLS) estimates of the corresponding risk premium for $T$ cross-sectional regressions, their t-statistics, and their p-values are presented in Table 5 for the models analyzed in this research. The results in the left-hand panel are obtained when using the return on the equally-weighted index to approximate the market return, while the findings associated with the value-weighted index are shown in the right-hand panel.

---

9 Similar results are obtained when we repeat the procedure dividing the total period into three subperiods, so that the number of available assets increases: 1982-1986 with 79 assets, 1987-1991 with 79, and 1992-1998 with 91 assets.
The first model estimated is the standard CAPM. As expected, given the previous empirical evidence from Spanish data, the results are not very satisfactory. The market risk premium, based on the equally-weighted index, is positive but quite variable, with a low t-statistic implying that the coefficient is not (statistically) different from zero.\(^{10}\) When we use the value-weighted index, Shanken’s (1992) adjustment for the standard error of the estimates produces even lower t-statistics. They are not reported, although they are available upon request.

### TABLE 5. Fama-MacBeth estimation.

<table>
<thead>
<tr>
<th></th>
<th>Panel A: equally-weighted index</th>
<th>Panel B: value-weighted index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\gamma_0) (\gamma_1) (\gamma_2) (\gamma_3)</td>
<td>(\gamma_0) (\gamma_1) (\gamma_2) (\gamma_3)</td>
</tr>
<tr>
<td>Standard CAPM: (R_{it} = \gamma_{0t} + \gamma_{1t} \beta_{1t} + u_{it})</td>
<td>0.8497 0.6584 1.6950 -0.2313</td>
<td>0.262 0.482 0.095 0.830</td>
</tr>
<tr>
<td>t-value</td>
<td>1.126 0.705</td>
<td>1.682 -0.214</td>
</tr>
<tr>
<td>p-value</td>
<td>0.262</td>
<td>0.830</td>
</tr>
<tr>
<td>Conditional CAPM: (R_{it} = \gamma_{0t} + \gamma_{mt} \beta_{mt} + \gamma_{bmt} \beta_{bmt} + \gamma_{dyt} \beta_{dyt} + u_{it})</td>
<td>0.3338 1.5059 -13.0052 -0.2529 0.2170 1.4824 -23.3483 -0.5828</td>
<td>0.429 1.415 -1.479 -0.910 0.215 1.433 -1.782 -1.382</td>
</tr>
<tr>
<td>t-value</td>
<td>0.429</td>
<td>1.415</td>
</tr>
<tr>
<td>p-value</td>
<td>0.668</td>
<td>0.159</td>
</tr>
<tr>
<td>Campbell model: ((R_{it} - R_f) = \gamma_{0t} + \gamma_{m1} \beta_{m1} + \gamma_{bmt} \beta_{bmt} + \gamma_{dyt} \beta_{dyt} + \gamma_{121} \beta_{121} + \gamma_{31} \beta_{31} + \gamma_{32} \beta_{32} + \gamma_{33} \beta_{33} + u_{it})</td>
<td>0.7266 2.075 -2.3485 0.0453 0.5733 0.5773 0.0655 0.0495</td>
<td>0.892 0.213 -1.309 0.667 0.632 0.605 0.022 0.514</td>
</tr>
<tr>
<td>t-value</td>
<td>0.892</td>
<td>0.213</td>
</tr>
<tr>
<td>p-value</td>
<td>0.374</td>
<td>0.193</td>
</tr>
<tr>
<td>Non linear pricing kernel: (R_{it} = \gamma_{0t} + \gamma_{m1} \beta_{m1} + \gamma_{bmt} \beta_{bmt} + \gamma_{dyt} \beta_{dyt} + \gamma_{121} \beta_{121} + \gamma_{31} \beta_{31} + \gamma_{32} \beta_{32} + \gamma_{33} \beta_{33} + u_{it})</td>
<td>1.5848 -3.9073 1.7381 -0.1619 1.6982 1.6907 0.1883 -0.0819</td>
<td>0.035 0.090 0.007 0.201 0.061 0.492 0.580 0.297</td>
</tr>
<tr>
<td>t-value</td>
<td>2.129</td>
<td>1.707</td>
</tr>
<tr>
<td>p-value</td>
<td>0.374</td>
<td>0.831</td>
</tr>
<tr>
<td>Fama and French model: ((R_{it} - R_f) = \gamma_{0t} + \gamma_{m1} \beta_{m1} + \gamma_{m2} \beta_{m2} + \gamma_{m3} \beta_{m3} + \gamma_{m4} \beta_{m4} + \gamma_{121} \beta_{121} + \gamma_{31} \beta_{31} + \gamma_{32} \beta_{32} + \gamma_{33} \beta_{33} + \gamma_{34} \beta_{34} + u_{it})</td>
<td>0.1631 0.5722 0.2298 -0.3987 0.4058 0.3600 0.1467 -0.3278</td>
<td>0.200 0.665 0.737 -0.524 0.445 0.367 0.461 -0.439</td>
</tr>
<tr>
<td>t-value</td>
<td>0.842</td>
<td>0.007</td>
</tr>
<tr>
<td>p-value</td>
<td>0.598</td>
<td>0.496</td>
</tr>
<tr>
<td>Multi-factor model from asymptotic principal components: ((R_{it} - R_f) = \gamma_{0t} + \gamma_{m1} \beta_{m1} + \gamma_{m2} \beta_{m2} + \gamma_{m3} \beta_{m3} + \gamma_{m4} \beta_{m4} + \gamma_{121} \beta_{121} + \gamma_{31} \beta_{31} + \gamma_{32} \beta_{32} + \gamma_{33} \beta_{33} + \gamma_{34} \beta_{34} + u_{it})</td>
<td>0.3626 0.340 0.1144 0.8872</td>
<td>0.529 0.439 0.131 0.683</td>
</tr>
<tr>
<td>t-value</td>
<td>0.598</td>
<td>0.896</td>
</tr>
<tr>
<td>p-value</td>
<td>0.598</td>
<td>0.496</td>
</tr>
</tbody>
</table>

This table presents the estimates by the two-step cross-sectional Fama-MacBeth procedure for the models indicated in each block. The dependent variable is the monthly return on size-deciles from 1982:1 to 1998:12. The explanatory variables are the betas for the different factors, and they are estimated with the 60 previous monthly returns to each cross-sectional estimation. The risk premium estimates are the average of the coefficient estimates from the monthly cross-sectional regressions, and they are reported as percentages. Their t-values are calculated using the standard deviation of the series of estimates.

The first model estimated is the standard CAPM. As expected, given the previous empirical evidence from Spanish data, the results are not very satisfactory.\(^{10}\) The market risk premium, based on the equally-weighted index, is positive but quite variable, with a low t-statistic implying that the coefficient is not (statistically) different from zero.\(^{11}\) When we use the value-weighted index, Shanken’s (1992) adjustment for the standard error of the estimates produces even lower t-statistics. They are not reported, although they are available upon request.

\(^{10}\) See Rubio (1988) and Sentana (1995) for relevant examples.

\(^{11}\) Shanken’s (1992) adjustment for the standard error of the estimates produces even lower t-statistics. They are not reported, although they are available upon request.
ed index the results are no better. Thus, the CAPM (not surprisingly) does not seem to do well for Spanish data in our sample period.

The next model estimated is the conditional CAPM. In this model, betas and expected returns are allowed to change throughout the business cycle. Recall that this is possible because we incorporate predictive factors such as the aggregate $BM$ and $DY$ ratios. The explanatory variables of the cross-sectional regression (betas) were obtained with previously available data using the traditional time-series regressions. We decided to jointly estimate the betas for the two instruments in a single time series regression because of the high correlation between them.

$$R_{it} = \alpha_{it} + \beta_{it} R_{mt} + e_{it}, \quad \forall t = t - 60, t - 59, ..., t - 1$$

$$R_{it} = a_{it} + \beta_{it}^{BM} BM_{t-1} + \beta_{it}^{DY} DY_{t-1} + e_{it}, \quad \forall t = t - 60, t - 59, ..., t - 1$$

The cross-sectional results are reported in the second block of Table 5. Although none of the risk premiums are significant at the usual significance levels, we now observe a considerable increment in the slope of the market beta relative to the estimates reported in the unconditional CAPM. Using the equally-weighted index, the market risk premium goes up from 0.66% in the standard CAPM to 1.5% in this conditional version. This improvement is also observed when using the value-weighted market portfolio. In this case, we obtain a risk premium for the $BM$ instrument which is larger (in absolute terms) than the one from the regression using the equally-weighted market index and is statistically different from zero at a level of 0.10. The sign of this parameter is due to the negative forecasting relationship between this factor and the market return (Nieto, 2002).

The third model estimated is the multi-beta version of the intertemporal asset pricing model of Campbell (1993). First we estimate the residuals from the VAR system (equation (19)) and then we compute the betas as equation (20) indicates. From the third block, we see that none of the coefficients estimated are statistically significant. The slope of the beta associated with the $DY$ ratio is positive, in contrast with the results of the previous model. This change in its sign should not be surprising because of two practical considerations. Firstly, the explanatory variables in the conditional CAPM and in Campbell’s model are not the same. In the former they represent the sensitivity of returns with respect to the predictive factors, and here they measure the sensitivity of returns to the residuals of the factors. Secondly, the strong link between $BM$ and $DY$ may produce instability in the relationship of these variables with returns when both factors are considered at the same time. When we observe the results obtained with the value-weighted stock index, the results are as discouraging as before. We should remember that a state variable in an intertemporal model would be reasonable only if it is capable of forecasting returns. As it turns out, Nieto (2002) shows that the $BM$ and $DY$ ratios are only able to predict the equally-weighted index return.

---

12 We might think that the strong correlation between $BM$ and $DY$ is deducting efficiency from the estimates of the coefficients of the cross-sectional conditional CAPM. We have repeated this estimation using the $BM$ ratio and the residuals from a regression of $DY$ on $BM$ as conditioning factors. The slopes of market return and $BM$ ratio are similar to those reported in Table 5 and the premium for the risk associated with the $DY$ ratio is lower, with an even lower t-statistic.
Next, we estimate a static model which does not consider a specific utility function but permits a non linear SDF. As seen in the previous section, a given SDF is linked to preferences throughout moments of the return distribution. In the same way that a linear pricing kernel relates expected returns to the covariance with the return on aggregate wealth (CAPM), a cubic pricing kernel is consistent with a model in which agents have preferences over the first four moments (expected returns, covariance of the returns with the return on wealth, covariance of the returns with quadratic return on wealth and covariance between returns and the cubic return on wealth). The three betas of this model are estimated in independent time-series regressions. The results in block four of Table 5 show that it is the only model that produces risk premiums statistically different from zero, at least when using the equally-weighted market index. Here, the quadratic return on wealth is a significant factor; in other words, is positive with a p-value of 0.007. This coefficient, associated with decreasing absolute risk aversion, indicates agent preferences for positive skewness in the distribution of returns. In this case, the market risk premium is also significant at 10%, although, unfortunately, it is negative. This result is probably due to the strong correlation between the factors considered by the model. Finally, the intercept is significantly different from zero.13

We next analyze the Fama and French (1993) three-factor model. In the first stage, betas for the factors are estimated with the following time-series regression.14

\[ R_{it} - R_{ft} = \alpha_i + \beta_{it}^m (R_{mt} - R_{ft}) + \beta_{it}^{SMB} SMB_t + \beta_{it}^{HML} HML_t + \epsilon_t, \quad \forall t = t - 60, t - 59, ..., t - 1 \]

The results of this time-series estimation for each size portfolio in our sample are slightly different from those obtained by Fama and French (1993).15 The betas for the market factor are highly significant and close to one. The size factor is also associated with significant betas, and their magnitude falls when the size of the portfolio increases, reflecting the negative relationship between return and size evidenced in previous empirical papers. HML is the variable with the least explanatory ability. It presents a beta significantly different from zero only for portfolios 1, 2 and 8, at 5%, and for portfolio 6 at 10%. This is positive for some portfolios and negative for others without any particular pattern. Finally, the alpha coefficient is (statistically) irrelevant for all portfolios, suggesting a well-specified factor model.16 Furthermore, the \( R^2 \) of our regressions are high, around 85%, although slightly lower than those obtained by Fama and French.

Regarding the estimate of the gamma coefficients for the cross-sectional regressions (block five of Table 5), the relationship between portfolio returns and factor betas is positive for both the market and size factors, and negative for the book-to-market factor, according to the negative relationship observed between BM and returns. Unfortunately, however, they are not statistically different from zero. Thus,

---

13 None of the coefficients in (27) is statistically significant when we use the value-weighted index.
14 This multiple regression takes into account the fact that SMB and HML are correlated with the market excess return.
15 The result of this estimation is not reported but it is available from the author upon request.
16 It may be noted that Fama and French (1993) find some alphas statistically different from zero. This finding would suggest a rejection in a joint test of the intercepts. Unfortunately, the authors perform the joint test on the alphas only for a case that includes both stocks and bonds.
Despite the reasonable performance of the time-series factor model, the cross-sectional results reported suggest that the Fama and French model cannot adequately explain average returns. This is consistent with the results presented by Jagannathan and Wang (1996).

Lastly, we estimate the multi-factor model that uses three asymptotic principal components as proxies of the true (unobservable) risk factors. Since this model does not assume any risk factor, its results are presented at the bottom of Table 5. Betas are previously estimated in independent time-series regressions. As we can see, none of the proxies of systematic risk factors is statistically relevant in explaining the variation of cross-sectional returns. Previous empirical research (Connor and Korajczyk (1988) with American data, or Rubio (1995) with Spanish data) reports similar results.

The fact that none of the risk premiums in any of the models studied is statistically significant in explaining average returns is, to say the least, surprising. To investigate this finding in greater depth, we analyze the cross-sectional estimates of each risk premium at each moment of time. In Table 6, we present the number of estimates that are either positive or negative. The total number of estimates is 144 and 143 for static and dynamic models respectively. The number in parentheses is the number of estimates with each sign that are statistically significant.

### TABLE 6.
Number of cross-sectional estimates positives and negatives

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_0$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;0</td>
<td>&lt;0</td>
<td>&gt;0</td>
<td>&lt;0</td>
<td>&gt;0</td>
</tr>
<tr>
<td>CAPM</td>
<td>84</td>
<td>60</td>
<td>71</td>
<td>73</td>
</tr>
<tr>
<td>Cond. CAPM</td>
<td>74</td>
<td>69</td>
<td>73</td>
<td>70</td>
</tr>
<tr>
<td>Campbell model</td>
<td>83</td>
<td>60</td>
<td>63</td>
<td>80</td>
</tr>
<tr>
<td>Non linear</td>
<td>87</td>
<td>57</td>
<td>68</td>
<td>76</td>
</tr>
<tr>
<td>Fama and French</td>
<td>79</td>
<td>65</td>
<td>68</td>
<td>76</td>
</tr>
<tr>
<td>Principal Compon.</td>
<td>72</td>
<td>72</td>
<td>76</td>
<td>68</td>
</tr>
</tbody>
</table>

This table presents the number of cross-sectional estimates for each risk premium in each model that is either positive or negative. The total number of estimates is 144 and 143 for static and dynamic models respectively. The number in parentheses is the number of estimates with each sign that are statistically significant.

17 When analysing the cross-sectional risk premiums at each moment of time some curious seasonal patterns arise. In particular, we find a very similar pattern for the premium associated to SMB in the Fama and French model and the premium of the second asymptotic principal component. In both models, and contrary to the CAPM case, the January effect for the premium of market return disappears. However, it is the second asymptotic principal component and the SMB premium which present a positive sign in January in 10 cases out of 12 observations. Moreover, the number of significant estimates for January is the same in both models. This finding deserves future research.
Given the difficulty of drawing conclusions from the results in Tables 5 and 6, we next represent the fitted average returns graphically against the realized average returns of the 10 size-sorted portfolios to provide a visual impression of the relative empirical perform-

FIGURE 1a. Realized versus fitted returns. The figure shows the pricing errors for each of the 10 size portfolios for the six models indicated in each graph. The average fitted returns are generated using the Fama-MacBeth regressions in Table 5. With the exception of the model which uses principal components, the market return is approximated with the return on the equally-weighted index. For the Campbell model, the Fama and French model, and the model with principal components the returns are in excess of the risk free rate.
This is done in Figures 1a and 1b. If a model fits perfectly, all points in those figures will lie along the 45-degree line. As we can see, for the standard CAPM, the Fama and French model and the model using principal components, all points follow a relatively horizontal pattern.

**FIGURE 1b.** Realized versus fitted returns. The figure shows the pricing errors for each of the 10 size portfolios for the six models indicated in each graph. The average fitted returns are generated using the Fama-MacBeth regressions in Table 5. The market return is approximated with the return on the value-weighted index. For the Campbell model, the Fama and French model, and the model with principal components the returns are in excess to the risk free rate.
trajectory. Hence, the models offer a practically constant return for the 10 different risk portfolios. Moreover, although the intermediate portfolios are concentrated in the middle of the 45-degree line, these static models have very poor fits for both the highest and the lowest portfolio returns. The dynamic models and the non-linear pricing kernel show more dispersion but they are able to draw the points in an upward trajectory guided by the 45-degree line. This, of course, suggests some compensation for covariance risk.

From all these results we can only conclude that dynamic models seem to perform relatively better in explaining the cross sectional returns for the Spanish market. This is consistent with investors taking their investment decisions in a time-changing framework where new information is considered when forming their expectations. In this sense, it is not reasonable to directly replace consumption for wealth in their utility functions because their investment horizons contain more than one period and they are not independent. The reader may think that this observation is questionable when one realizes that the only model that offers significant risk premiums is the cubic (but static) pricing kernel CAPM. To give robustness to our conclusion, we estimate a conditional version of the non-linear CAPM. Given the results of Table 5, and in order to pick up the quadratic market return, we now use a second order Taylor expansion for the SDF, and we write the relationship between expected return and betas in conditional terms as in the conditional CAPM. Additionally, we assume that the changes in risk premiums are explained using one lag of the book-to-market aggregate ratio. The resulting model has, therefore, five betas: the covariances between returns and market return, quadratic market return, one-lag BM, quadratic one-lag BM, and the product of market return and one-lag BM. When we estimate the risk premiums of this model, the quadratic market return factor is now not significant and the parameter associated with the simple BM is statistically significant at 10%.18

5. TESTING THE SPECIFICATION IN SDF REPRESENTATIONS

In this section we again compare the six models analyzed before, but we now establish the relative performance of their different empirical specifications. To do this, we evaluate all models using their SDF representation and the Generalized Method of Moments (GMM) for estimation. Under this framework, there is a performance measurement which compares different models by imposing a pre-specified weighting matrix. A performance ranking is thus obtained.

The main advantage of this method of econometric evaluation of asset pricing models is its generality, it can be used for linear and non-linear models, unlike the classical beta method, which needs returns and factors to be jointly normally distributed in order to make non-linear models linear. The question is whether estimates of factor risk premiums are more efficient and

18 Now the multicollinearity problem is avoided because the five factor betas have been estimated with a unique time-series regression.
specification tests more powerful in the SDF model than in the beta model. Using Monte Carlo simulations, Jagannathan and Wang (2002) find that asymptotically, as well as in finite samples, both methods provide equally precise estimates.

### 5.1. Estimation and testing

Consider a model in which the SDF is a function of a set of factors, like all the models studied in this research:

$$E \left[ \hat{R}_t \left( \delta_0 + \delta_1 R_1^t + \ldots + \delta_K R^K_t \right) \right] = 1$$  \hspace{1cm} (36)

The difference between models is given by the alternative factors that the term inside brackets considers. This term contains unknown parameters \((\delta_k)\), which can be estimated by demanding that the pricing errors implied by this SDF be as low as possible relative to market prices (Hansen (1982)).

Let \(\tilde{R}_t\) be the N-dimensional vector of gross returns at time \(t\), \(\gamma_t\) is the \((K+1)\) vector where the first element is one and the rest are the \(K\) risk factors at moment \(t\), and \(\delta\) is the \((K+1)\) vector of parameters.

\[
\begin{pmatrix}
\tilde{R}_{t1} \\
\tilde{R}_{t2} \\
\vdots \\
\tilde{R}_{tN}
\end{pmatrix}
\begin{pmatrix}
1 \\
\gamma_1 \\
\vdots \\
\gamma_K
\end{pmatrix}
\begin{pmatrix}
\delta_0 \\
\delta_1 \\
\vdots \\
\delta_K
\end{pmatrix}
\]

(37)

The GMM gives an estimate of \(\delta\) so that the expectation of the pricing errors of the model should be zero. Therefore, we should choose \(\delta\) to minimize the following quadratic form:

$$g' \hat{W} g$$  \hspace{1cm} (38)

where \(g = E(f_i(\hat{\delta}))\), \(f_i(\hat{\delta}) = \hat{R}_t \delta \gamma_t' - 1^N\) and \(1^N\) is an N-dimensional vector of ones.

It can be proved that the resultant estimate \(\hat{\delta}\) has the following asymptotic distribution:

$$\sqrt{T} (\hat{\delta} - \delta) \overset{d}{\longrightarrow} N(0, V)$$  \hspace{1cm} (39)

where \(V = \left( \frac{\partial f_i}{\partial \delta} \right) \hat{W} \left( \frac{\partial f_i}{\partial \delta} \right)^{-1} \left( \frac{\partial f_i}{\partial \delta} \right) \hat{W} \left( \frac{\partial f_i}{\partial \delta} \right)^{-1} \left( \frac{\partial f_i}{\partial \delta} \right) \hat{W} \left( \frac{\partial f_i}{\partial \delta} \right)\) and \(S = E(f_i(\delta)f_i(\delta))\).

Hansen and Singleton (1982) show that the optimal weighting matrix \((W)\) is the inverse of the variance of the pricing errors, which means that this matrix minimizes the asymptotic variance of the estimator. Given the optimal weighting matrix, this value multiplied by the sample size has asymptotically a \(\chi^2\) distribution with \(N-K\) degrees of freedom. Of course, we know that this procedure produces (asymptotically) efficient and consistent estimators. However, the test involves some inconveniences. First, the statistic favors models with highly variable errors, because it is
inversely related to their variance. Secondly, this statistic cannot be used to compare the specification of alternative models because it depends on a weighting matrix that is different across them. As a solution, Hansen and Jagannathan (1997) propose the use of the second-moment matrix of returns in place of the traditional (optimal) weighting matrix. They advocate a weighting matrix which depends exclusively on the dependent variables but not on the explanatory variables. In particular, they suggest the inverse of the covariance matrix of returns,

\[ W = \left[ E(\tilde{R}_t \tilde{R}_t') \right]^{-1} \quad (40) \]

This is the matrix we will use in this section. The GMM estimators obtained with this weighting matrix are consistent but not efficient. However, the value of the minimized function represents, as before, a performance measure which now allows the researcher to establish a relative ranking of performance among competing models: the higher this value, the worse the model performs. This measure is known as the Hansen-Jagannathan distance, and is given by the following expression,

\[ Dist = \sqrt{g(\hat{\delta})'Wg(\hat{\delta})} \quad (41) \]

The statistic for the specification test is

\[ HJ = T(Dist)^2 \quad (42) \]

Since the \( W \) matrix is generally not optimal, the statistic proposed does not have a \( \chi^2 \) distribution. Jagannathan and Wang (1996) show that this statistic is asymptotically distributed as a weighted sum of (N-K) independent \( \chi^2 \) random variables. In practice, we obtain the asymptotic distribution of the statistic by simulating 500 samples of these (N-K) variables \( \chi^2 \) with size \( T \).

### 5.2. Empirical results

In this section we present the GMM estimates of the SDF representation of the models described in section 3, and the result of the performance test using the statistic based on the Hansen-Jagannathan distance.

In Table 7, the GMM estimates of the coefficients in the moment restriction of each model, their t-statistics and the corresponding p-values are presented. In the right hand column (referred to as \( HJ \)), the pricing errors measurement based on the Hansen-Jagannathan weighting matrix is reported. Its p-value appears under this statistic obtained, as explained earlier, from the simulated distribution of a linear combination of N-K independent \( \chi^2 \) variables. The sample size is 144 observations (January 1987 – December 1998) for all tests except for the Conditional CAPM and for the Campbell Model, in which the one-lagged variables are used. The number of (N-K) independent chi-square variables is the number of assets (in our case 10 portfolios) minus the number of parameters of the model (number of factors plus one). The market factor is the equally-weighted return of all assets in the sample.

In general terms, as in the Fama-MacBeth estimation, we find that none of the factors in any of the models produces a statistically significant coefficient, although we cannot reject any of the models when we employ the \( HJ \) statistic. In a comparative analysis presented by Jagannathan
and Wang (1996), the conditional CAPM is the only model that cannot be rejected using this measure. With respect to Campbell’s model, Nieto (2002), using the GMM procedure with the optimal weighting matrix, shows that both predictive factors have risk premiums statistically different from zero, although her sampling period is not the same as in this paper. The results for the non-linear pricing kernel confirm the evidence provided by Dittmar (2002) in the sense that none of the coefficients are statistically significant but $\delta_2$ has the largest value.\(^{19}\)

In spite of the similarities in the estimations of the six models, there are results in Table 7 that allow us to conclude in favor of the dynamic models relative to the static ones. First, the

\(^{19}\) Although in the work of Dittmar (2002) it is observed that incorporating human capital considerably improves the performance of this nonlinear kernel.
FIGURE 2. Realized versus fitted gross returns. The figure shows the pricing errors for each of the 10 size portfolios for the six models indicated in each graph. The average fitted returns are generated using the GMM estimates in Table 6. With the exception of the model which uses principal components, the market return is approximated with the return on the equally-weighted index.

intercept in static models is clearly relevant indicating that the average cross-sectional gross returns for the 10 portfolios according to these models are constant. This observation agrees with the pattern shown in Figures 1a and 1b. Second, the $HJ$ statistic confirms the relatively better performance of the dynamic models. It has a value of around 10 for static models such as the CAPM, the non linear CAPM, the Fama and French model and the model with asymptotic principal components, with p-values between 11% and 24%. However, the $HJ$ distance is considerably lower for the conditional CAPM and Campbell’s model, with p-values that indicate that
the average difference between the observed return and the one generated by these models is zero approximately 95-97% of times.\(^{20}\)

To support the observation that the conditional CAPM and the intertemporal model of Campbell have lower average pricing errors as indicated by the \(HJ\) distance, we again present graphs of fitted returns based now on the regressions of Table 7 versus realized returns. The result of this exercise is found in Figure 2. In this case, the better fit of the two dynamic models is clearly evidenced. As the \(HJ\) statistic indicates, the pricing errors generated by Campbell’s model are insignificant. The model is able to fit all ten realized returns, the extreme ones included, and it produces a maximum error of 0.001 for portfolio 7.

Before concluding, we want to mention some contradictions that can be observed in comparing the results of the two empirical approaches presented here. First, there is a theoretical relationship between the risk premiums of the beta specification models and the parameters of the SDF (see equation (11) for an example relative to the CAPM), which is not captured by the estimation. Second, although the risk premiums are not statistically significant in most of the models when we use the Fama and MacBeth methodology, we cannot reject any of them with the \(JH\) test.

Regarding the first fact, Jagannathan and Wang (2002) prove that the risk premiums estimated in the two different specifications are very similar and have the same asymptotic variance. However, our empirical analysis differs from theirs in two aspects. Firstly, they estimate beta model representations using GMM, which produces just one estimation of each factor beta for the whole sample period and betas are obtained jointly with the risk premiums. We run the Fama-MacBeth procedure where betas are estimated at each point in time. As Cochrane (2001) points out, the estimation of a single beta using time-series averages and the Fama-MacBeth estimate do not have to be identical if the explanatory variables vary over time. Secondly, the information involved in the two method implementations is different because in the Fama-MacBeth procedure we use 60 previous observations for the estimation of time-series betas.

With respect to the fact that none of the models is rejected using \(JH\) distance, we might think that the statistical power of the tests to reject the null is substantially low. For example, our sample period is not large relative to studies employing US data. Previous simulation studies find that standard errors given by GMM procedure are likely to be understated in small samples (Smith, 1994). Therefore, the statistic for individual significance tends to reject the hypothesis that the parameters are equal to zero too often and, likewise, the specification test tends to reject the model too often. Here we run into the opposite case entirely, and hence we have not considered applying finite sample adjustment. In addition, we have sought to prove the robustness of our results using complementary analysis. We find that consistent results are obtained both when we test the specification of the models using the Hansen and Singleton (1982) optimal weighting matrix in the GMM implementation, and when we use a Wald test based on the quadratic sum of the residuals from OLS or Generalized Least Squares estimation of the beta models.\(^{21}\) Lastly, the results may also be due to low levels of dispersion among portfolio

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\(^{20}\) The \(HJ\) statistic for the conditional version of the non linear CAPM has a value of 4.55 instead of 9.53 for the static model, with a \(p\)-value of 87%.

\(^{21}\) As an example, when we test the specification of the CAPM, the model that is most likely to be rejected given empirical literature, the value of a Chi-square statistic based on Generalized Least Squares estimation is 11.68 with a \(p\)-value of 0.17.
returns. To check for robustness in portfolio formation, the analysis has been repeated using alternative criteria for ranking stocks. The results (not shown but available upon request) are very similar when we use portfolios based on book-to-market ratio, momentum strategies, spreads or market betas.

6. CONCLUSION

Due to the weak empirical support obtained by the unconditional CAPM, alternative sources of common risk factors have been proposed in the asset pricing literature. In this paper we have chosen five asset pricing models which, to our understanding, can be considered as representative candidates of the ones employed in this research framework. The conditional CAPM of Jagannathan and Wang (1996) and the intertemporal asset pricing model of Campbell (1993) are employed to pick up changing business cycles, in which book-to-market and dividend yield ratios are chosen as predictors of changes in the investment opportunity set. A non-linear pricing kernel version of the static CAPM has the potential to explain some of the observed nonlinearities in the data related to preference for positive skewness. The static three-factor model proposed by Fama and French (1993) allows for other (non-identified) sources of systematic risk in addition to market return. Finally, a general multi-factor APT, where the unobservable factors are estimated by the asymptotic principal component technique, is also included in our tests.

The results of the cross-sectional regressions of the returns of ten size-sorted portfolios on the betas for the factors considered do not show substantial differences among the alternative models analyzed, and they are not as satisfactory as we would like them to be. The empirical evidence shows that characteristics such as size or book-to-market ratio can be correlated with stock returns (Fama and French (1992) or Kothari et al. (1995)), but asserting that these characteristics constitute the basis of systematic risk factors is not trivial. In the estimation of the Fama and French model using the Fama-MacBeth procedure, we verify that none of the mimicking portfolios of size and book-to-market represent risks that explain the cross-sectional average returns. It is true that our time series is not excessively long and that it refers to a very particular market, but Jagannathan and Wang (1996) obtain results that corroborate our findings with US market data. Likewise, these results question the use of the Fama and French model as a way of adjusting for risk in event studies or in the evaluation of investment funds, among others. The model with factors estimated by the asymptotic principal components technique does not offer good results either, and in the estimation of the non-linear CAPM we must remark on the significant role of the quadratic market return.

With respect to dynamic models within the Fama-MacBeth procedure, the empirical results are not much better than the findings for static models. The results only show that BM is significant in the conditional CAPM when the return on wealth is approximated with the value-weighted market index. However, an important feature is the larger magnitude of the market risk premium that these models obtain with respect to the static ones. In the Jagannathan and Wang (1996) paper, the market risk premium is also more relevant when the time-varying beta is included because of the conditional character of the model. The market factor may not be as irrelevant as history shows, but it may just be that a static model imposes excessively unrealistic assumptions on its beta.
Lastly, an estimation of the SDF representation of the models by GMM and a comparative analysis using the Hansen-Jagannathan distance is performed. Again, none of the factors in the GMM estimation are statistically significant, maybe because eight of the ten portfolios present very little dispersion in returns, as we can observe in Figures 1a and 1b. In fact, this low cross-sectional variability in returns may also be responsible for the impossibility of rejecting any model using the HJ distance. In any case, however, it seems clear that dynamic models are adding some information which reduces pricing errors with respect to static models (in the same way as the estimation of market premium improves when state variables are considered). The HJ value is considerably smaller for Campbell’s intertemporal asset pricing model and the conditional CAPM of Jagannathan and Wang (1996) than for static models. This result is perfectly manifested in the graphical representation of the pricing errors in Figure 2. This estimation methodology, which weights the pricing errors of the ten portfolios with the variability of their returns, fits the realized returns as well as the evidence shown for the dynamic models in Figure 2. We definitively recommend dynamic asset pricing models relative to static competitors when explaining average returns on the Spanish stock market.

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